MATH 1050 Sample Final Exam

The following three formulas may be helpful in solving problems from question 1.

- Sum of the first *n* terms of an arithmetic series with first term a_1 and common difference *d*:

$$\frac{n}{2}(2a_1 + (n-1)d)$$

- Sum of the first *n* terms of a geometric series with first term a_1 and common ratio *r*:

$$\frac{a_1(1-r^n)}{1-r}$$

- Sum of an infinite geometric series:

$$\sum_{n=1}^{\infty} a\,r^{n-1} = \frac{a}{1-r}$$

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1. Evaluate each of the following, if possible. If the series diverges, explain why.

$$\sum_{k=1}^{50}(2k-1)$$

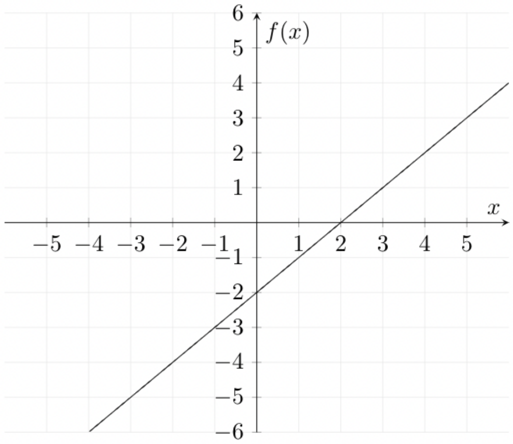
$$\sum_{j=1}^{\infty} 1.5(2)^{j-1}$$

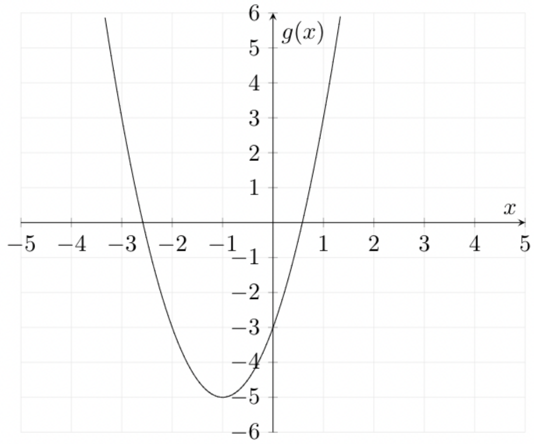
$$\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)^{k-1}$$

2. Given that x=2 is a zero of the polynomial function $f(x)= x^3 -2x^2 +4x -8$, find all the other real and complex roots of f(x).

3. Given $f(x)= x^3$, complete the following tasks:
   a.) Graph f(x) and label three points.
   b.) List the transformations (in order) for the graph of f(x) to obtain the graph of $g(x)= -(x-1)^3 -2$.
   c.) Graph g(x) and label three points.

4. The graphs of functions $f(x), g(x)$ are below. Use them to compute the indicated values.





a.) $(f \circ g)(1)$ b.) $(g \circ f)(2)$ c.) $(g \circ g)(0)$ d.) $(f \circ f)(-1)$

5. Solve the following logarithmic equations. Eliminate any extraneous solutions.

a.) $\log_4 x + \log_4(x-6) = 2$ b.) $\log_3 4x - \log_3(x+2) = 1$

6. Evaluate: a.) $\ln e^4$ and b.) $\log_{10}\left(\frac{1}{100}\right)$

7. Write as a single logarithm: $2\log_4 x + 5\log_4 y - \frac{1}{2}\log_4 z$.

8. Solve the inequality $\frac{4}{x+1} \geq 1$ and give your answer in interval notation.

9. For the function $f(x) = x^2 - 2x$,
a.) Find the *x*-intercepts, if any exist.
b.) Find the *y*-intercept, if it exists.
c.) Find the vertex.
d.) Sketch a graph of $y = f(x)$.

10. Find the inverse of the function $f(x) = (x-1)^3 - 2$.

11. Classify each of the following sequences as arithmetic, geometric, or neither.

a.) $17, 9, 1, -7, -15, \ldots$

b.) $2, -4, 8, -16, 32, \ldots$

c.) $\frac{8}{3}, 4, 6, 9, \frac{27}{2}, \ldots$

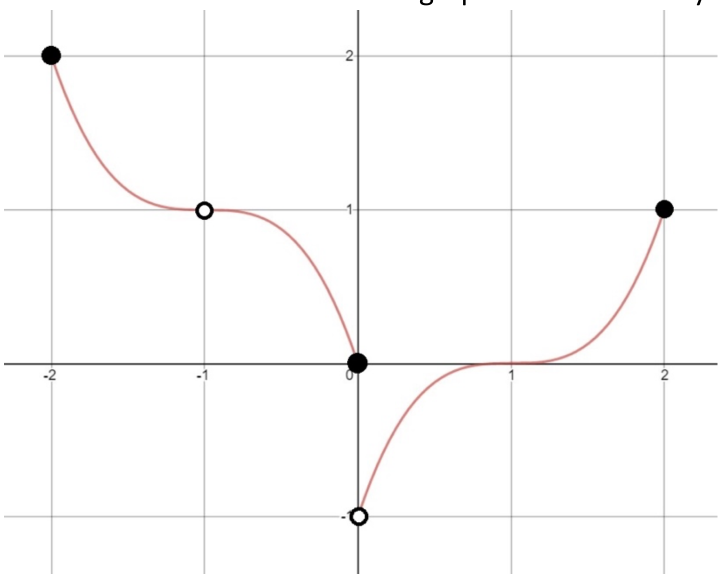
d.) $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, \ldots$

e.) $-2, 6, -10, 14, -18, \ldots$

f.) $3^2, 3^4, 3^6, 3^8, \ldots$

12. Find a formula for the 90th term of the sequence $-7, -4, -1, 2, 5, \ldots$

13. Is the function shown in the graph invertible? Why or why not?



Solutions to MATH 1050 Sample Final Exam

1.) Using the first formula from the exam,

$$\sum_{k=1}^{50}(2k-1) = \frac{50}{2}\left(2(1)+49(2)\right)$$
$$= \frac{50}{2}(100)$$
$$= 50(50)$$
$$= 2{,}500$$

$\sum_{j=1}^{\infty} 1.5(2)^{j-1}$ diverges since $2 \geq 1$.

Using the third formula from the exam,

$$\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)^{k-1} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

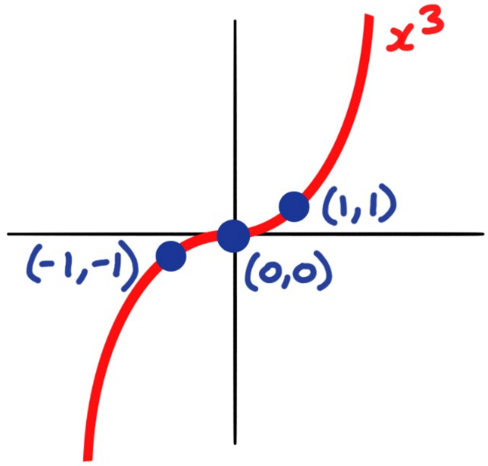
2.) Since 2 is a zero of x^3-2x^2+4x-8, $x-2$ is a factor of x^3-2x^2+4x-8. Using long division:

$$\begin{array}{r|l}
 & x^2+4 \\
\hline
x-2 & x^3-2x^2+4x-8 \\
 & \underline{x^3-2x^2} \\
 & \qquad\qquad 4x-8 \\
 & \qquad\qquad \underline{4x-8} \\
 & \qquad\qquad\quad 0
\end{array}$$

so x^3-2x^2+4x-8 equals $(x-2)(x^2+4)$.

$x-2$ has 2 as a root, and the roots of x^2+4 are $2i$ and $-2i$, so the roots of x^3-2x^2+4x+8 are $2, 2i$, and $-2i$.

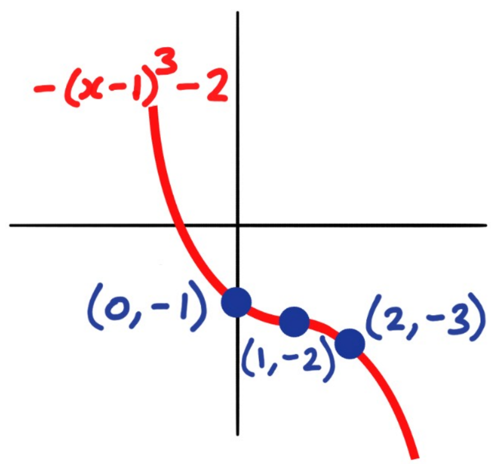
3.) a.)



b.) $x^3 \longmapsto \underbrace{-}_{\substack{\uparrow \\ \text{flip over} \\ x\text{-axis}}}(x\underbrace{-1}_{\substack{\uparrow \\ \text{right 1}}})^3 \underbrace{-2}_{\substack{\nwarrow \\ \text{down 2}}}$

To graph $g(x) = -(x-1)^3 - 2$, start with the graph of $f(x) = x^3$, shift it right 1, flip over x-axis, and shift down 2.

c.)



4.) a.) $f(g(1)) = f(3) = 1$

b.) $g(f(2)) = g(0) = -3$

c.) $g(g(0)) = g(-3) = 3$

d.) $f(f(-1)) = f(-3) = -5$

5.) a.) $2 = \log_4(x) + \log_4(x-6)$

$= \log_4(x(x-6))$

$= \log_4(x^2-6x)$

so $4^2 = x^2-6x$. Thus, $x^2-6x-16=0$.

The roots of $x^2-6x-16$ are 8 and -2, however, -2 is an extraneous solution since if $x=-2$, then $\log_4(x) = \log_4(-2)$ and we can't take a logarithm of a negative number.

b.) $1 = \log_3(4x) - \log_3(x+2) = \log_3\left(\frac{4x}{x+2}\right)$

so $3^1 = \frac{4x}{x+2}$. Thus, $3(x+2) = 4x$,

so $6 = x$.

6.) $\ln(e^4) = 4$ and

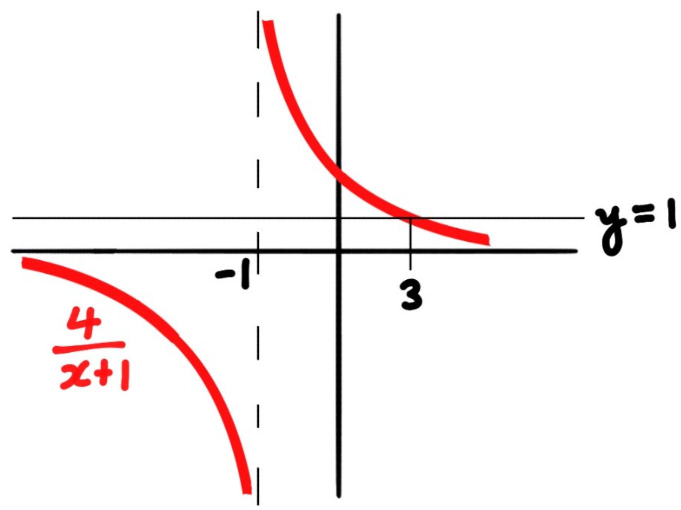
$\log_{10}\left(\frac{1}{100}\right) = \log(10^{-2}) = -2$.

7.) $2\log_4 x + 5\log_4 y - \frac{1}{2}\log_4 z$

$$= \log_4 x^2 + \log_4 y^5 - \log_4 \sqrt{z}$$

$$= \log_4\left(\frac{x^2 y^5}{\sqrt{z}}\right)$$

8.) $\frac{4}{x+1}$ is $1/x$ shifted left by 1, and scaled vertically by 4:



$\frac{4}{x+1} \geq 1$ where the graph of $\frac{4}{x+1}$ is at or above $y=1$. $\frac{4}{x+1}$ is at $y=1$ where $\frac{4}{x+1}=1$, so $x+1=4$ and $x=3$.

From the graph, we see the answer is $(-1,3]$.

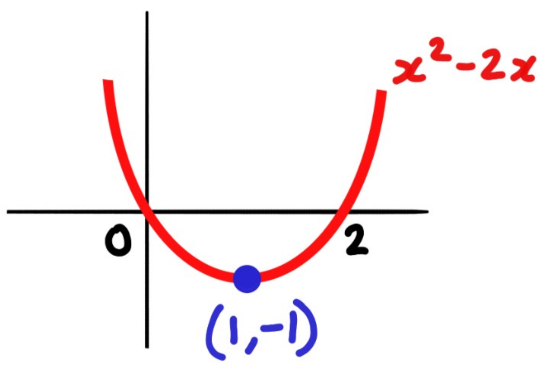
9.) a.) $x^2 - 2x = x(x-2)$ has roots 0 and 2.

These are the x-intercepts.

b.) The y-intercept is $(0)^2 - 2(0) = 0$.

c.) The vertex is at $x = 1$, the midpoint of the roots. Thus, $y = (1)^2 - 2(1) = -1$ so the vertex is at $(x, y) = (1, -1)$.

d.)



10.) Replace $f(x) = (x-1)^3 - 2$ with

$x = (f^{-1}(x) - 1)^3 - 2$, and solve for $f^{-1}(x)$:

$x + 2 = (f^{-1}(x) - 1)^3$, so $\sqrt[3]{x+2} = f^{-1}(x) - 1$,

and $f^{-1}(x) = \sqrt[3]{x+2} + 1$.

11.) a.) Arithmetic, subtracting 8.

b.) Geometric, multiplying -2.

c.) Geometric, multiplying $3/2$.

d.) Arithmetic, subtracting $\frac{2}{2}$.

e.) Neither

f.) Geometric, multiplying 3^2.

12.) $-7 + 90(3) = 263$

13.) It's not invertible. It doesn't pass the horizontal line test.