

**Question 1.** Suppose  $G$  is a group.

1. Suppose  $N$  is a normal subgroup of  $G$  and  $G$  is finitely generated. Then the quotient group  $G/N$  is also finitely generated.
2. If  $N$  is normal in  $G$  with both  $N$  and  $G/N$  finitely generated. Then  $G$  is finitely generated.
3. If  $H \leq G$  a subgroup of a group  $G$ , with  $H$  finite index, then  $H$  finitely generated if and only if  $G$  is. The only if is easier to prove using geometry!

**Question 2.** Suppose  $G$  is a finitely generated group. Show that for any two finite (may as well assume symmetric) generating sets  $S, T$ , that  $C(G, S)$  is QI to  $C(G, T)$ .

**Question 3.** Show that the composition of two QI's is a QI and that QI is an equivalence relation on metric spaces.

**Question 4.** Verify what I said about  $T_3$  and  $T_4$  being QI and show that  $T_m$  is QI to  $T_n$  for any  $m \neq n$  both greater than or equal to 3.

**Question 5.** Suppose  $\phi : \Gamma_1 \rightarrow \Gamma_2$  is a homomorphism between finitely generated groups. Show that if  $\phi$  is a quasi-isometric embedding then  $\ker(\phi)$  is finite and that  $\phi$  is a quasi-isometry if and only if  $\ker(\phi)$  and  $\Gamma_2/\text{im}(\phi)$  are both finite.

**Question 6.** Think about why  $F_2$  and  $\mathbb{Z}_2$  cannot be quasi-isometric. I am not asking you to write a proof of this - you need some machinery to write a real proof - but can you "explain" why they shouldn't be?