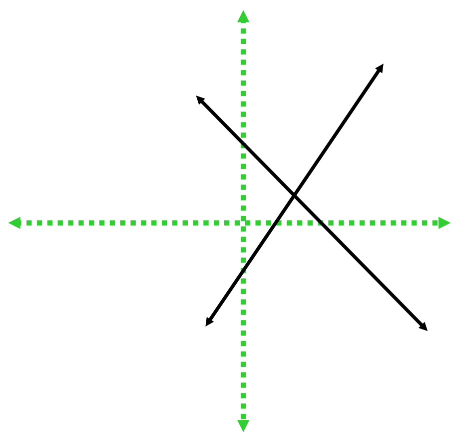
MATH 1010 ~ Intermediate Algebra

Chapter 4: SYSTEMS OF EQUATIONS

Section 4.1: SYSTEMS OF EQUATIONS

Objectives:

- Determine if ordered pairs are solutions of systems of equations.
- Solve systems of equations graphically
- Solve systems of equations by substitution.
- Use systems of equations to model and solve real life problems.



$x + y = 3$
$x - y = -2$

Vocabulary:

system of linear equations : a collection of eqns; typically we will have the same of eqns as variables

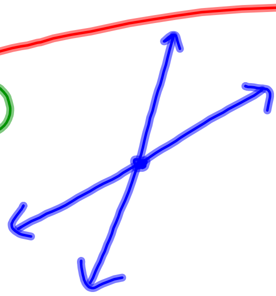
solution

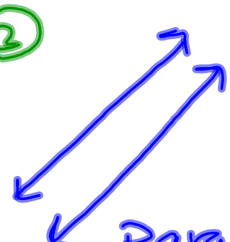
for a system of eqns, the solution is the pt. of intersection

point of intersection

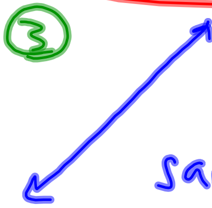
is soln	no soln	same line
consistent	inconsistent	dependent

independent (not same lines)

① 

② 

parallel lines

③ 

same line

independent consistent

independent inconsistent

dependent consistent

Three methods to solve a system of equations:

1. Graphing: graph both curves on same axes and find the pt. of intersection
(least used)

2. Substitution: use one of the eqns to solve for one variable; substitute that into the other eqn.

3. Elimination

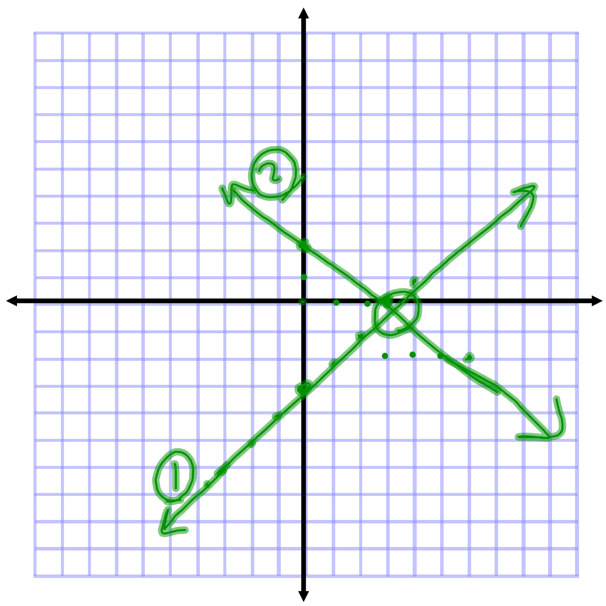
① EXAMPLE:

Solve each system by graphing

a) ① $x - y = 3$
② $2x + 3y = 7$

① $x = y + 3$
$y = x - 3$

② $3y = -2x + 7$
$y = \frac{-2}{3}x + \frac{7}{3}$

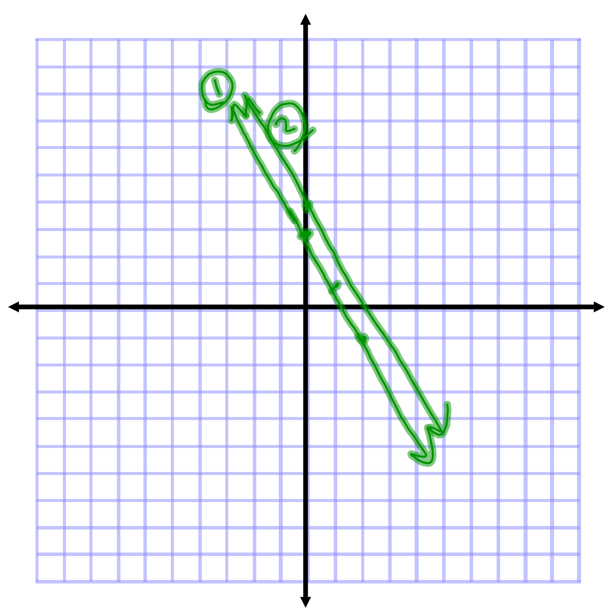


b) ① $2x + y = 3$
② $2y = -4x + 8$

① $y = -2x + 3$

② $y = -2x + 4$

N.S.



② EXAMPLE

Solve by substitution

a) ① $y = 2x + 1$
② $3x + 2y = 16$

$1 \to 2$ ② $3x+2(2x+1)=16$

$3x+4x+2=16$

$7x+2=16$

$-2 \quad -2$

$7x=14$

$x=2$

① $y=2(2)+1$

$=4+1$

$=5$

$\boxed{\text{soln: } (2,5)}$

b) ① $x + y = 3$
② $\frac{2y}{2} = \frac{2x + 6}{2}$

$\Leftrightarrow$ ② $y=x+3 \rightarrow$ ① $x+(x+3)=3$

$2x+3=3$

$-3 \quad -3$

$\frac{2x}{2}=\frac{0}{2}$

$x=0$

② $y=0+3$

$y=3$

$\boxed{\text{soln: } (0,3)}$

c) ① $2x + 5y = 15$
② $y = -2/5\ x$

① $2x+\cancel{5}\left(\frac{-2}{\cancel{5}}x\right)=15$

$2x+-2x=15$

$0 \neq 15$

$\Rightarrow$ N.S. (no soln)
(parallel lines)

a) ① $x - y = 5$
② $2x = 2y + 10$

$\overset{+y}{} \quad \overset{+y}{}$

$\Rightarrow$ ① $x = 5 + y$

$\to$ ② $2(5+y) = 2y + 10$

$10 + 2y = 2y + 10$

$-10 \qquad -10$

$2y = 2y$

$-2y \quad -2y$

$0 = 0$

$\Rightarrow$ same line

b) ① $y = -3/2\ x + 4$
② $3x + 2y = 3$

② $3x + 2\left(\frac{-3}{2}x + 4\right) = 3$

$\cancel{3x} + \cancel{-3x} + 8 = 3$

$8 \neq 3$

$\Rightarrow$ N.S.

④ EXAMPLE:

Set up a set of equations and solve these problems.

a) ① The sum of two numbers is 160.
② The larger number is three times the smaller number.

$x = 1^{st}$ number , $y = 2^{nd}$ number (larger)

$$\begin{cases} ① & x + y = 160 \\ ② & y = 3x \end{cases}$$

① $x + 3x = 160$

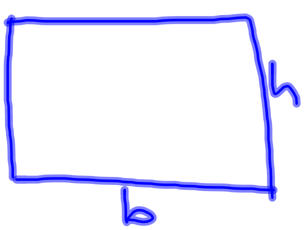
$4x = 160$

$x = 40$

② $y = 3(40)$

$= 120$

b) The perimeter of a rectangle is 90 meters.
The length is 1½ times the width.
Find the dimensions of the rectangle.



$P = 2b + 2h = 90$ ①

$b = \frac{3}{2}h$ ②

① $2\left(\frac{3}{2}h\right) + 2h = 90$

$3h + 2h = 90$

$\frac{5h}{5} = \frac{90}{5}$

$h = 18$

② $b = \frac{3}{\cancel{2}}(\cancel{18}^{9})$

$= 27$

$b = 27$ m

$h = 18$ m

c) Ten pounds of a nut mixture sells for \$6.95 per pound. The mixture is made from two kinds of nuts; peanuts at \$5.65 per pound and cashews at \$8.95 per pound.

How many pounds of each will be used in the mixture?

	rate \$/lb	# lbs	\$
P	5.65	x	5.65x
C	8.95	y	8.95y
mix	6.95	10	6.95(10)

① $x+y=10 \iff x=10-y$

② $5.65x+8.95y=6.95(10)$

② $5.65(10-y)+8.95y=69.5$

$56.5-5.65y+8.95y=69.5$

$56.5+3.30y=69.5$

$-56.5 \qquad -56.5$

$(10)\ 3.3y=13\ (10)$

$33y=130$

$y=$ 3.94 lbs cashews

① $x \simeq 10-3.94$

$=$ 6.06 lbs peanuts