MATH 1010 ~ Intermediate Algebra

Chapter 6: RATIONAL EXPRESSIONS, EQUATIONS AND FUNCTIONS

Section 6.1: Rational expressions and functions

Objectives:

- Find the domain of a rational function.
- Evaluate a rational function.
- Simplify rational expressions.

$$\frac{9x-2}{3x+1} \qquad \frac{3-2x^2}{5x^2}$$

Vocabulary

A Rational Function : a function that is a fraction w/ polynomial numerator & denominator

$f(x) = \frac{h(x)}{g(x)}$, $h(x), g(x)$ polynomials

Domain

set of allowable inputs
(all x-values)

① Example

Find the domain for these. (we can't divide by zero)

a) $f(x) = \frac{3}{x-1}$

domain: $x \neq 1$, $x \in \mathbb{R}$

$(-\infty, 1) \cup (1, \infty)$

b) $g(x) = \frac{4x-2}{6}$ (polynomial)

$= \frac{4x}{6} - \frac{2}{6}$

domain: $x \in \mathbb{R}$

c) $y = \frac{3x-2}{(x-3)(x+2)}$

domain: $x \in \mathbb{R}$, $x \neq 3, -2$

$(x-3)(x+2) \neq 0$

$x-3 \neq 0$ & $x+2 \neq 0$

$x \neq 3$ $\quad$ $x \neq -2$

d) $h(x) = \frac{9x-2}{4x^2+1}$

domain: $x \in \mathbb{R}$

② EXAMPLE

Evaluate these.

a) $f(-2) \quad when \quad f(x) = \dfrac{x^2 - 3x}{x - 4}$

$$f(-2) = \frac{(-2)^2 - 3(-2)}{-2-4} = \frac{4+6}{-6} = \frac{10}{-6} = \frac{5}{-3}$$

b) $g(1) \quad when \quad g(x) = \dfrac{x-3}{2x+1}$

$$g(1) = \frac{1-3}{2(1)+1} = \frac{-2}{2+1} = \frac{-2}{3}$$

③ EXAMPLE

Simplify these.

$2 \cdot -12 = -24$

	$2x$	3
x	$2x^2$	$3x$
-4	$-8x$	-12

a) $\dfrac{2x^3-3x}{6x^2}$

$= \dfrac{\cancel{x}(2x^2-3)}{6x^{\cancel{2}}}$

$= \dfrac{2x^2-3}{6x}$

b) $\dfrac{2x^2-5x-12}{-3x+12}$

$= \dfrac{(2x+3)\cancel{(x-4)}}{-3\cancel{(x-4)}}$

$= \boxed{\dfrac{2x+3}{-3}, \; x \neq 4}$

c) $\dfrac{x^2-16}{x^2-2x-8}$

$= \dfrac{\cancel{(x-4)}(x+4)}{(x+2)\cancel{(x-4)}}$

$= \boxed{\dfrac{x+4}{x+2}, \; x \neq 4}$

d) $\dfrac{2x^2+2xy-4y^2}{5x^3-5xy^2}$

$= \dfrac{2(x^2+xy-2y^2)}{5x(x^2-y^2)}$

$= \dfrac{2(x+2y)\cancel{(x-y)}}{5x\cancel{(x-y)}(x+y)}$

$= \boxed{\dfrac{2(x+2y)}{5x(x+y)}, \; x \neq y}$