MATH 1010 ~ Intermediate Algebra

Chapter 6: RATIONAL EXPRESSIONS, EQUATIONS AND FUNCTIONS

Section 6.3: Adding and Subtracting Rational Expressions

Objectives:

- Add rational expressions.
- Subtract rational expressions.

$$\frac{2}{3x} + \frac{3}{5x} \qquad \frac{2x}{3-x} - \frac{3x}{5+x}$$

Adding Rational Expressions With Like Denominators

$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

$\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

a) $\dfrac{16+x}{5x} - \dfrac{11-x}{5x} =$

$$\frac{16+x-(11-x)}{5x}$$

$$= \frac{16-11+x+x}{5x}$$

$$= \boxed{\frac{5+2x}{5x}}$$

b) $\dfrac{3}{3x-2} + \dfrac{5x}{3x-2}$

$$= \boxed{\frac{3+5x}{3x-2}}$$

Adding or Subtracting if the Denominators are Different

a) $\frac{6}{7x}+\frac{-2}{5x}$

LCD = 35x

$= \frac{6}{7x}\left(\frac{5}{5}\right)+\frac{-2}{5x}\left(\frac{7}{7}\right)$

$= \frac{30}{35x}+\frac{-14}{35x}$

$= \frac{30+-14}{35x} = \frac{16}{35x}$

b) $\frac{2}{(x-2)}+\frac{3}{(x+1)}$

LCD = (x-2)(x+1)

$= \frac{2}{(x-2)}\left(\frac{x+1}{x+1}\right)+\frac{3}{(x+1)}\left(\frac{x-2}{x-2}\right)$

$= \frac{2(x+1)+3(x-2)}{(x-2)(x+1)}$

$= \frac{2x+2+3x-6}{(x-2)(x+1)}$

$= \frac{5x-4}{(x-2)(x+1)}$

c) $\frac{3x}{x^2-9}+\frac{5}{3-x}$

$=\frac{3x}{(x-3)(x+3)}+\frac{5}{-(x-3)}$ $\quad 3-x=-x+3=-(x-3)$

$LCD=(x-3)(x+3)$

$=\frac{3x}{(x-3)(x+3)}+\frac{-5}{(x-3)}$

$=\frac{3x}{(x-3)(x+3)}+\frac{-5}{(x-3)}\left(\frac{x+3}{x+3}\right)$

$=\frac{3x-5(x+3)}{(x-3)(x+3)}$

$=\frac{3x-5x-15}{(x-3)(x+3)}=\boxed{\frac{-2x-15}{(x-3)(x+3)}}$

d) $\frac{2x-5}{6x+9}-\frac{4}{2x^2+3x}+\frac{1}{x}$

$LCD=x(2x+3)(3)=3x(2x+3)$

$=\frac{2x-5}{3(2x+3)}-\frac{4}{x(2x+3)}+\frac{1}{x}$

$=\frac{(2x-5)}{3(2x+3)}\left(\frac{x}{x}\right)-\frac{4}{x(2x+3)}\left(\frac{3}{3}\right)+\frac{1}{x}\left(\frac{3(2x+3)}{3(2x+3)}\right)$

$=\frac{x(2x-5)-12+3(2x+3)}{3x(2x+3)}$

$=\frac{2x^2-5x-12+6x+9}{3x(2x+3)}$

$=\frac{2x^2+x-3}{3x(2x+3)}=\frac{(2x+3)(x-1)}{3x(2x+3)}=\boxed{\frac{x-1}{3x},\ x\neq -3/2}$

e) $\dfrac{x}{x^2+15x+50}+\dfrac{7}{x+10}-\dfrac{x-1}{x+5}$

LCD = $(x+10)(x+5)$

$$=\frac{x}{(x+5)(x+10)}+\frac{7}{(x+10)}-\frac{x-1}{(x+5)}$$

$$=\frac{x}{(x+5)(x+10)}+\frac{7}{(x+10)}\left(\frac{x+5}{x+5}\right)-\frac{(x-1)}{(x+5)}\left(\frac{x+10}{x+10}\right)$$

$$=\frac{x+7(x+5)-(x-1)(x+10)}{(x+10)(x+5)}$$

$$=\frac{x+7x+35-(x^2-x+10x-10)}{(x+10)(x+5)}$$

$$=\frac{x+7x+35-x^2-9x+10}{(x+10)(x+5)}=\frac{-x^2-x+45}{(x+10)(x+5)}$$