

Section 7.5: Solving Radical Equations

Objective:

- * Solve a radical equation.

$$\sqrt{2x-1} - \sqrt{3x} = 2$$

TRUE or FALSE?

If $u = v$, then $u^n = v^n$.

True

Strategy for Solving Radical Eqn

- ① Isolate one of the radical expressions on one side of eqn.
- ② Raise both sides of eqn to the power that will undo the root.
- ③ Finish solving.

- ① EXAMPLE
Solve for the variable.

WARNING: check answers for all even root eqns.

a) $\sqrt{x} - 5 = 0$

$$(\sqrt{x}) = (5)$$

$$x = 25$$

b) $\sqrt{x^2 + 5} - 3 = 0$

$$(\sqrt{x^2 + 5}) = (3)$$

$$x^2 + 5 = 9$$

$$x^2 = 4$$

$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

$$x-2=0 \quad \vee \quad x+2=0$$

$$x=2, \quad x=-2$$

c) $\sqrt{3y+1} - \sqrt{y+15} = 0$

$$(\sqrt{3y+1}) = (\sqrt{y+15})$$

$$3y+1 = y+15$$

$$2y+1 = 15$$

$$2y = 14$$

$$y = 7$$

d) $(2\sqrt[3]{10-3x}) = (\sqrt[3]{2-x})$

$$2^3(10-3x) = 2-x$$

\equiv

$$8(10-3x) = 2-x$$

$$80 - 24x = 2 - x$$

$$80 = 2 + 23x$$

$$78 = 23x$$

$$x = 78/23$$

② EXAMPLE

Solve for the variable.

check: $\sqrt{\frac{13}{4}+3} - \sqrt{\frac{13}{4}-1} = \sqrt{\frac{25}{4}} - \sqrt{\frac{9}{4}}$
 $= \frac{5}{2} - \frac{3}{2} = 1 \checkmark$

a) $\sqrt{x+3} - \sqrt{x-1} = 1$
 $\quad \quad \quad +\sqrt{x-1} \quad +\sqrt{x-1}$

$(\sqrt{x+3})^2 = (1+\sqrt{x-1})^2$

$x+3 = (1+\sqrt{x-1})(1+\sqrt{x-1})$

$x+3 = \cancel{1} + \sqrt{x-1} + \sqrt{x-1} + \cancel{x-1}$

$\underset{-x}{x}+3 = 2\sqrt{x-1} + \underset{-x}{x}$

$\frac{3}{2} = \frac{2\sqrt{x-1}}{2}$

$(\frac{3}{2})^2 = (\sqrt{x-1})^2$

$\frac{9}{4} = x-1$
 $\quad +1 \quad +1$

$\frac{13}{4} = x$

check:

$\sqrt{\frac{1}{4}} + \sqrt{\frac{1}{4}+2} = \frac{1}{2} + \sqrt{\frac{9}{4}}$
 $= \frac{1}{2} + \frac{3}{2} = 2 \checkmark$

b) $\sqrt{x} + \sqrt{x+2} = 2$
 $\quad \quad \quad -\sqrt{x} \quad \quad \quad -\sqrt{x}$

$(\sqrt{x+2})^2 = (2-\sqrt{x})^2$

$x+2 = (2-\sqrt{x})(2-\sqrt{x})$

$x+2 = 4 - 2\sqrt{x} - 2\sqrt{x} + x$

$x+2 = 4+x-4\sqrt{x}$
 $\underset{-4-x}{-4-x} \quad \underset{-4-x}{-4-x}$

$\frac{-2}{-4} = \frac{-4\sqrt{x}}{-4}$

$(\frac{1}{2})^2 = (\sqrt{x})^2$

$\frac{1}{4} = x$

③ EXAMPLE

Solve for the variable.

a) $(x + 4)^{5/3} = 32$

$$\left((x+4)^{5/3} \right)^{3/5} = 32^{3/5}$$

$$x+4 = 32^{3/5}$$

$$x+4 = \sqrt[5]{32^3}$$

$$x+4 = \left(\sqrt[5]{32} \right)^3$$

$$x+4 = 2^3$$

$$x+4 = 8$$

$$x = 4$$

check: $(x+4)^{5/3} = 32$

if $x=4$, $(8)^{5/3} = (\sqrt[3]{8})^5$
 $= 2^5 = 32 \checkmark$

b) $\sqrt{x} = x - 6$

$$(\sqrt{x})^2 = (x-6)^2$$

$$x = (x-6)(x-6)$$

$$x = x^2 - 6x - 6x + 36$$

$$x = x^2 - 12x + 36$$

$$0 = x^2 - 13x + 36$$

$$0 = (x-4)(x-9)$$

$$x-4=0 \text{ or } x-9=0$$

$$x=4 \text{ or } x=9$$

check $\sqrt{x} = x-6$

~~$x=4$~~ : $\sqrt{4} \stackrel{?}{=} 4-6$

$2 \stackrel{?}{=} -2$ NO

$x=9$: $\sqrt{9} \stackrel{?}{=} 9-6$

$3 \stackrel{?}{=} 3$ \checkmark