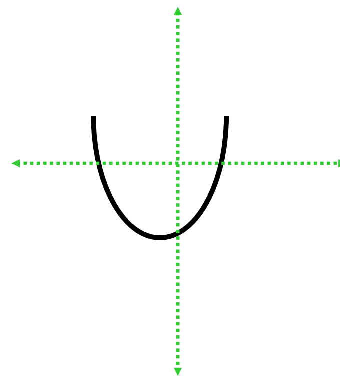
Chapter 8.4: Graphing Quadratic Functions

Objectives:

- ✯ Determine the vertex of a parabola by completing the square or finding the x-intercepts.
- ✯ Sketch a parabola.
- ✯ Given a graph, write the equation of the parabola.
- ✯ Use this information in application problems.

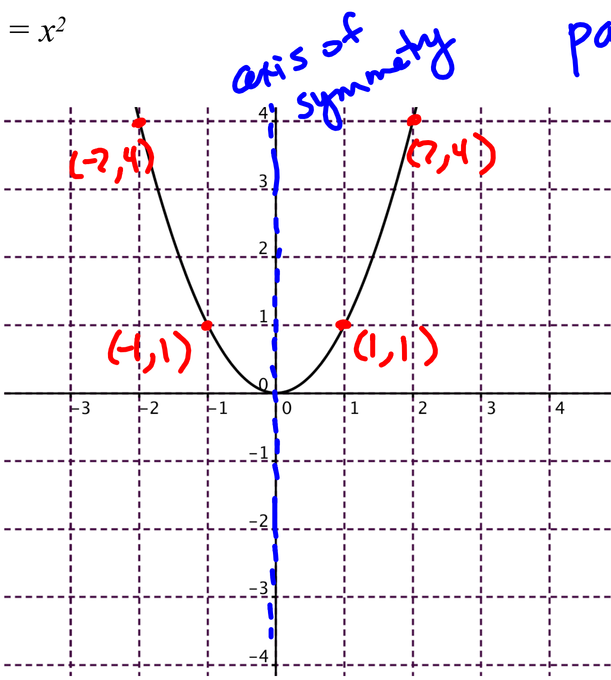
$$3(x+1)^2 - 5 = y$$



The graph of the basic quadratic function looks like this.

$y = x^2$

parabola, concave up



*Key items to note:

*Vertex at (0,0)

*Axis of symmetry
(x=0) y-axis

*Key symmetric points on the left and right of the vertex.

also $y=x^2$ is an even function

Transformations to the graph from $y = x^2$ to
$\boxed{y = a(x-h)^2 + k}$ (standard form)

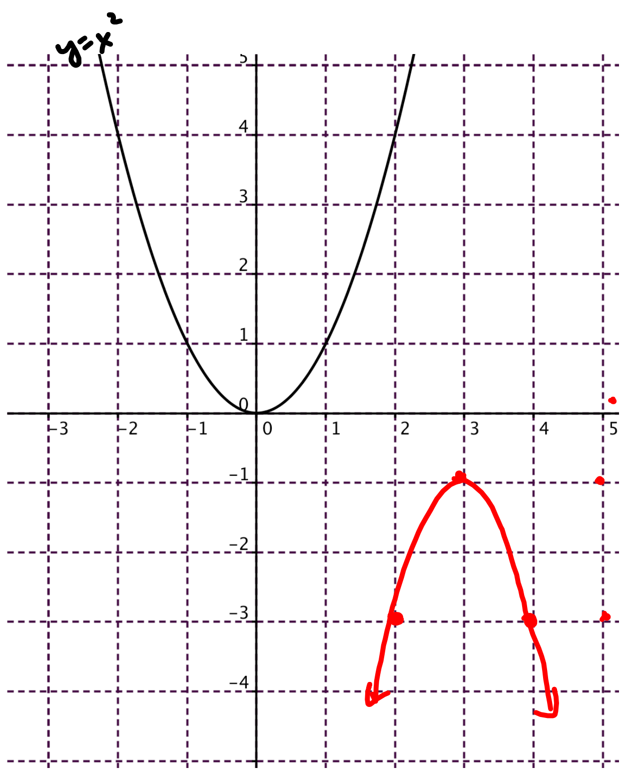
Stretch $|a|$ if $|a| > 1$, then vert. stretch (skinnier parabola)
if $|a| < 1$, then vert. shrink (wider parabola)

Reflect vert reflection if $a < 0$

Shift (h, k) is the new vertex

$y = -2(x-3)^2 - 1$

base $y = x^2$	vert stretch $y = 2x^2$	vert reflection $y = -2x^2$	right shift $y = -2(x-3)^2$	shift down $y = -2(x-3)^2-1$
(0,0)	(0, 0)	(0, 0)	(3, 0)	(3, -1)
(-1,1)	(-1, 2)	(-1, -2)	(2, -2)	(2, -3)
(1,1)	(1, 2)	(1, -2)	(4, -2)	(4, -3)



- vertex (3, -1)
- concave down parabola
- axis of symmetry $x = 3$

Two ways to graph a quadratic function are:

1. If it is in factored form
   a. Find the x-intercepts.
   b. Find the x-value halfway between the x-intercepts. This will be the x-value of the vertex.
   c. Determine the y-value of the vertex.
   d. Plot the vertex and intercepts.

$y = (x-3)(x+1)$

x-intercepts occur when $y=0$

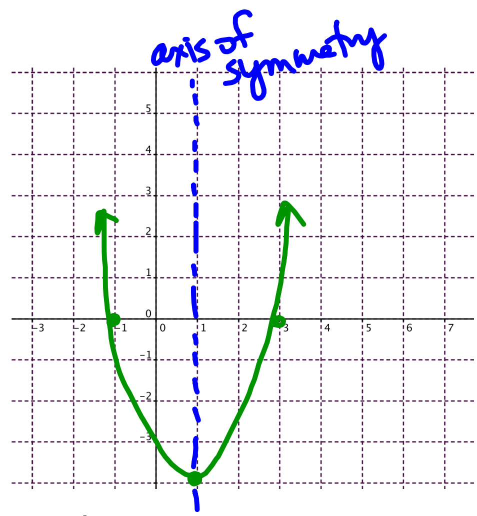
$0=(x-3)(x+1)$

$x-3=0$ or $x+1=0$

$x=3$ or $x=-1$ $\Rightarrow (3,0)$ $(-1,0)$

vertex $(1,-4)$

$y=(1-3)(1+1)$
$=-2(2)=-4$



2. If it is not factorable or you prefer not to factor it
   a. Complete the square to put it in standard form.
   b. Plot the vertex.
   c. Plot the symmetric points 1 unit to the left and right of the vertex.

$y = x^2+6x+5$

$\left(\frac{6}{2}\right)^2 = 3^2 = 9$

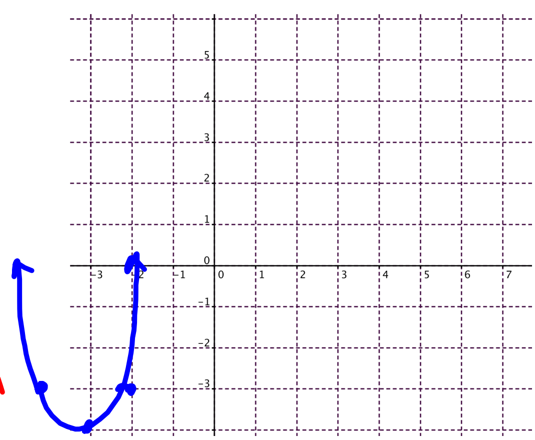
$y=(x^2+6x+9)+5-9$

$y=(x+3)^2+-4$

$\Rightarrow$ vertex $(-3,-4)$

no stretching
no reflection

$(-2,-3)$
$(-4,-3)$



Ex 1: Find the vertex of this parabola by completing the square, then, sketch the parabola.

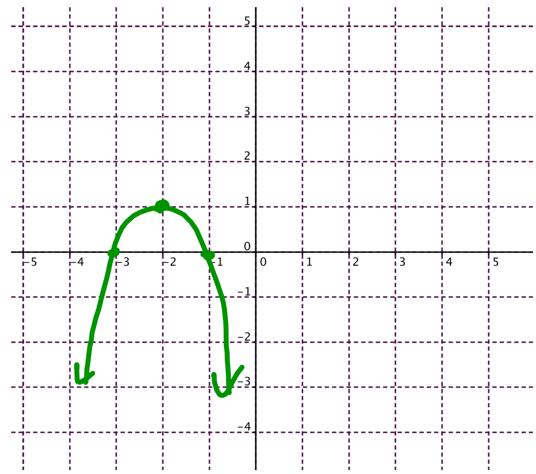
$y = f(x) = -x^2 - 4x - 3$

$y = -(x^2 + 4x + 4) - 3 + 4$

$y = -(x+2)^2 + 1$

* vertex $(-2, 1)$

* vert. reflection



EX 2: Find the vertex of this parabola by factoring, then sketch it.

$f(x) = x^2 + 4x - 5$

$y = (x+5)(x-1)$

X-intercepts:

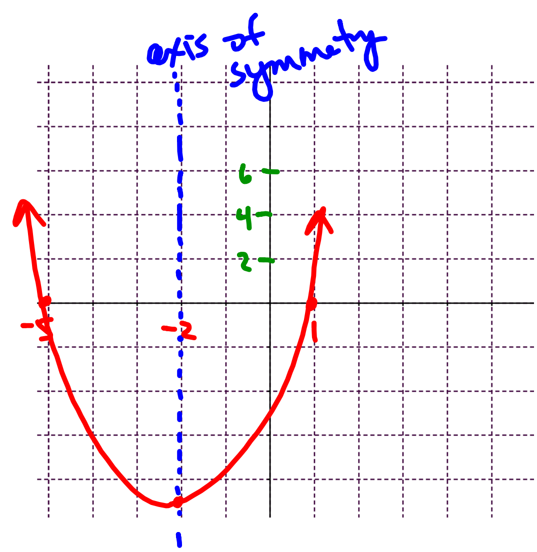
$0 = (x+5)(x-1)$

$x+5=0$ or $x-1=0$

$x=-5$ or $x=1$

$(-5, 0)$ $(1, 0)$

$\Rightarrow$ vertex $(-2, -9)$



$y = (-2+5)(-2-1) = 3(-3) = -9$

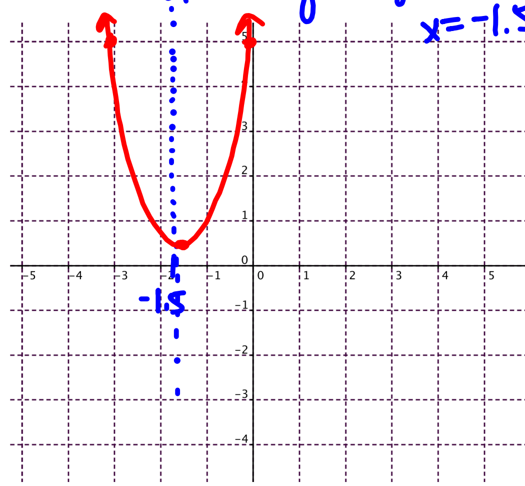
Ex 3: Use symmetry to find the vertex of this parabola, then sketch it.

Hint: Find the y-intercept, then find the symmetric point at which it intersects with the line y = 5. Use these two points to determine the vertex.

$f(x) = 2x^2 + 6x + 5$

coefficient of x^2 is positive $\Rightarrow$ concave up

axis of symmetry at $x=-1.5$



y-intercept occurs when $x=0$:

$y = 0+0+5 \Rightarrow (0,5)$

Due to symmetry, there's exactly one other pt on parabola w/ same y-value.

$5 = 2x^2+6x+5$

$-5 \qquad\qquad -5$

$\Rightarrow$ vertex at $(-1.5, ?)$

$0 = 2x^2+6x$

$0 = 2x(x+3)$

$x=0$ or $x+3=0$

$x=-3$

$\Rightarrow$ parabola goes through $(-3,5)$

when $x=-1.5$,

$y = 2(-1.5)^2+6(-1.5)+5$

$= 2(2.25)-9+5$

$= 4.5-9+5$

$= -4.5+5$

$= 0.5$

$\Rightarrow$ vertex $(-1.5, 0.5)$

Ex 4: A child launches a toy spaceship from their treehouse. The height of the rocket is given by the function, $h(x) = -\frac{1}{4}x^2 + 5x + 9$, where x is the horizontal distance in feet from the base of the tree.

a) Determine the height from which the spaceship is launched.

launch occurs when $x = 0$ ft.

$h = 0 + 0 + 9 = 9$ ft.

max occurs at vertex

b) What is the maximum height the rocket attains?

Since leading coefficient is negative, we have concave down parabola.

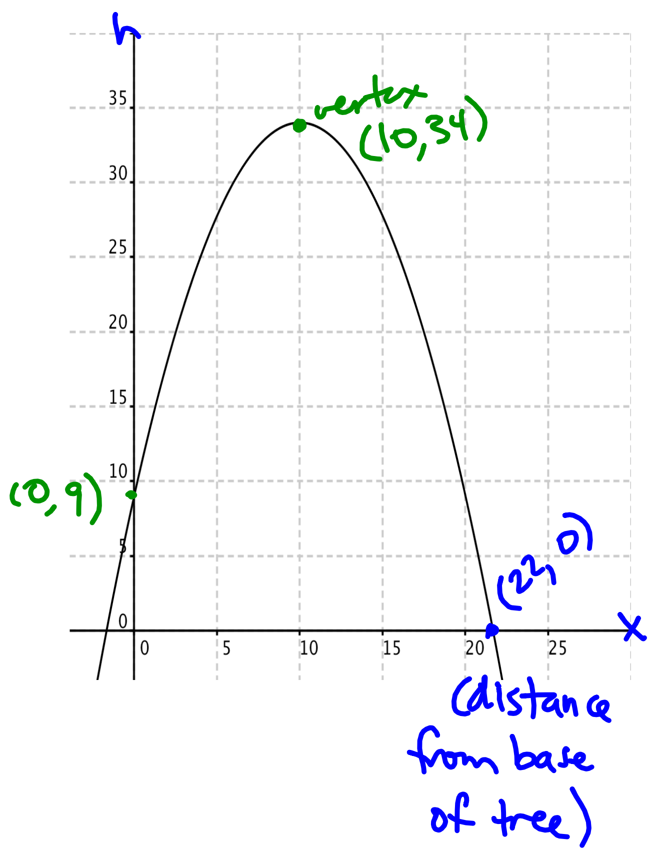
$\Rightarrow$ max ht is $h = 34$ ft.

c) How far from the base of the tree where it is launched does the rocket land? (Assume flat ground around the tree.)

landing occurs when $h = 0$ ft.

at that pt

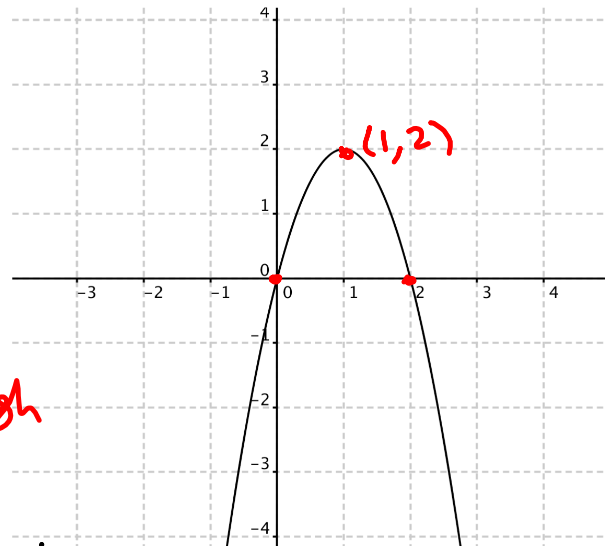
$x = 22$ ft.



Ex 5: Write an equation for this function in two different forms,

General: $y = ax^2+bx+c$

Standard: $y = a(x-h)^2+k$



vertex at (1,2)

$\boxed{y = -2(x-1)^2+2}$

(check that this goes through (0,0) and (2,0))

to get general form, multiply this out.

$y = -2(x^2-2x+1)+2$

$y = -2x^2+4x-2+2$

$\boxed{y = -2x^2+4x}$

because

$(x-1)^2 = x^2-x-x+1$

$= x^2-2x+1$

Also note, x-intercepts at (0,0) and (2,0)

$\Rightarrow y = a(x-0)(x-2)$

$y = a(x)(x-2)$ and $a=-2$ for this graph

$\boxed{y = -2x(x-2)}$ (factored form)