MATH 1010 ~ Intermediate Algebra

Chapter 9: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Section 9.2: Composite and Inverse Functions

Objectives:

- Form compositions of two functions and find the domain.
- Use the Horizontal Line Test to determine whether a function has an inverse.
- Verify that two functions are inverses.
- Find inverse functions algebraically.

$$(f \circ g)(x)$$

$$f^{-1}(x)$$

Composition of Two Functions

(nested functions; function of a function)

$$f(g(x)) \neq g(f(x))$$

notation

$$(f \circ g)(x) = \underline{f(g(x))}$$

composition sign

read "f composed with g of x" or "f of g of x"

ex $f(x) = 2x^2 + 3$ $\qquad g(x) = x - 9$

$f(\heartsuit) = 2\heartsuit^2 + 3$ $\qquad g(\heartsuit) = \heartsuit - 9$

$(f \circ g)(x) =$

$f(g(x))$

$= f(x-9)$

$= 2(x-9)^2 + 3$

$= 2(x-9)(x-9) + 3$

$= 2(x^2 - 18x + 81) + 3$

$= 2x^2 - 36x + 162 + 3$

$= 2x^2 - 36x + 165$

$(g \circ f)(x) =$

$g(f(x))$

$= g(2x^2+3)$

$= (2x^2+3) - 9$

$= 2x^2 - 6$

① EXAMPLE

Find the compositions. State the domain where applicable.

domain: $x \in \mathbb{R}$ domain: $x \in \mathbb{R}$

$f(x) = \sqrt[3]{x-1}$ $g(x) = 3x^2 + 2$

(domain)

$a)$ $(g \circ f)(x) = g(f(x)) = g(\sqrt[3]{x-1})$

$= 3(\sqrt[3]{x-1})^2 + 2$

domain: $x \in \mathbb{R}$

$= 3(x-1)^{2/3} + 2$

$b)$ $(f \circ g)(5) = f(g(5)) = f(3(5^2)+2)$

$= f(75+2) = f(77)$

$= \sqrt[3]{77-1} = \sqrt[3]{76}$

$c)$ $(g \circ f)(-2) = g(f(-2)) = g(\sqrt[3]{-2-1}) = g(\sqrt[3]{-3})$

$= 3(\sqrt[3]{-3})^2 + 2$

$= 3\sqrt[3]{9} + 2$

② EXAMPLE

Evaluate these.

$$f(x) = x^3 - 1 \qquad g(x) = 2x + 5$$

a) $(f \circ g)(0) = f(g(0)) = f(2(0)+5)$

$= f(5)$

$= 5^3 - 1 = 125 - 1 = \boxed{124}$

$(g \circ f)(2)$

b) $g(f(2)) = g(2^3 - 1) = g(7) = 2(7) + 5 = 14 + 5 = 19$

An Inverse Function: ① requirement 1: inverse for a function (it has to be a fn)

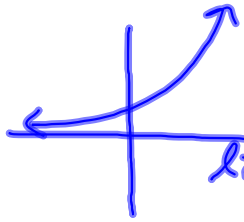
② it passes horizontal line test

Defn function has exactly one output for every input (passes vertical line test)

Horizontal Line Test

fn

does not have inverse



passes vert. & horiz. line tests

notation: $\boxed{g(x) = f^{-1}(x) \quad \underline{iff} \quad f(g(x)) = g(f(x)) = x}$

$f^{-1}(x)$ read as "f inverse of x"

inverse fn undoes the base fn

Verify that these are inverse functions.

$$f(x) = 4x^3 - 5 \qquad g(x) = \sqrt[3]{\frac{x+5}{4}}$$

if they are inverses, then $f(g(x)) = g(f(x)) = x$

$$f(g(x)) = f\left(\sqrt[3]{\frac{x+5}{4}}\right) = 4\left(\sqrt[3]{\frac{x+5}{4}}\right)^3 - 5$$

$$= \cancel{4}\left(\frac{x+5}{\cancel{4}}\right) - 5$$

$$= x + \cancel{5} - \cancel{5} = x$$

$$g(f(x)) = g(4x^3 - 5)$$

$$= \sqrt[3]{\frac{(4x^3 - \cancel{5}) + \cancel{5}}{4}}$$

$$= \sqrt[3]{\frac{\cancel{4}x^3}{\cancel{4}}} = \sqrt[3]{x^3} = x \quad \checkmark$$

$g(x) = f^{-1}(x)$

③ EXAMPLE

Find the inverse of each function if it exists.

Algebraic Method to find inverse function

① switch the x & y

② solve for y; the remaining fn is inverse fn.

a) $f(x) = 2x^5 - 1 = y$

① $2y^5 - 1 = x$

$\frac{2y^5}{2} = \frac{x+1}{2}$

$y^5 = \frac{x+1}{2}$

$\Rightarrow \boxed{y = \sqrt[5]{\frac{x+1}{2}} = f^{-1}(x)}$

b) $g(x) = x^2 + 1$

① $x = y^2 + 1$

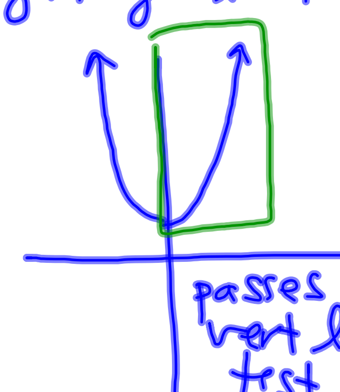
② $x - 1 = y^2$

$y = \pm\sqrt{x-1}$ < $\sqrt{x-1}$, $-\sqrt{x-1}$

$g^{-1}(x)$ DNE

$\boxed{h(x) = x^2 + 1,\ x \geq 0}$ $\boxed{h^{-1}(x) = \sqrt{x-1}}$

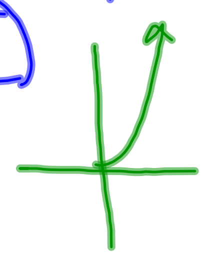
$g(x) = y = x^2 + 1$



passes vert line test

fails horiz. line test

$\Rightarrow$ doesn't have inverse



c) $h(x) = x^3 - 1$

① $x = y^3 - 1$

② $x + 1 = y^3$

$y = \boxed{\sqrt[3]{x+1} = h^{-1}(x)}$