MATH 1010 ~ Intermediate Algebra

Chapter 9: EXPONENTIAL AND LOGARITHMIC FUNCTIONS

Section 9.4: Properties of Logarithms

Objectives:

- Use the properties of logarithms to evaluate logarithms.
- Use the properties of logarithms to rewrite, expand and condense logarithmic expressions.

$$\log_2(xy)=\log_2 x+\log_2 y$$

$$\ln(x^2)=2*\ln x$$

Properties of Logarithms

1. $\log_a(uv) = \log_a u + \log_a v$
2. $\log_a\left(\dfrac{u}{v}\right) = \log_a u - \log_a v$
3. $\log_a u^n = n\log_a u$

NOTE: A $\log x = \log_{10} x$

(no base, then it's base 10)

B $\ln x = \log_e x$

WARNING

- logarithm does NOT distribute

$\log(x+y) \neq \log x + \log y$

- as a word, "log" does not divide out

ex $\dfrac{\log 5}{\log 3} \neq \dfrac{5}{3}$

- "log" is NOT multiplied by anything

ex $\log 5 \neq \log \cdot 5$

① EXAMPLE

Evaluate or simplify these expressions.

$a)\quad \ln(e^2 \cdot e^4) = \ln(e^6) \underset{③}{=} 6 \ln e = 6$

$\ln e = ? \iff e^{?} = e$

$? = 1$

$b)\quad \log_6 2 + \log_6 3$

$\underset{①}{=} \log_6(2 \cdot 3) = \log_6 6 = 1$

$c)\quad \log_2 5 - \log_2 40 \underset{②}{=} \log_2\left(\frac{5}{40}\right) = \log_2\left(\frac{1}{8}\right)$

$= \log_2\left(2^{-3}\right) = -3$

$d)\quad \ln\left(\frac{6}{e^5}\right)$

$\underset{②}{=} \ln 6 - \ln e^5$

$= \ln 6 - 5$

WARNING:

$\ln 6 - 5$

$\neq \ln(6-5)$

$\ln 6 - 5 = (\ln 6) - 5$

$= \ln(6) - 5$

② EXAMPLE

Expand these expressions using the properties of logarithms.

$$a) \quad \ln(5x) \overset{①}{=} \ln 5 + \ln x$$

$$b) \quad \log_5 \sqrt{xy} = \log_5 (xy)^{1/2} \overset{③}{=} \frac{1}{2}\log_5(xy)$$

$$\overset{①}{=} \frac{1}{2}\left[\log_5 x + \log_5 y\right]$$

$$c) \quad \log\sqrt{\frac{3x}{x-5}} = \log\left(\frac{3x}{x-5}\right)^{1/2} \overset{③}{=} \frac{1}{2}\log\left(\frac{3x}{x-5}\right)$$

$$\overset{②}{=} \frac{1}{2}\left(\log(3x) - \log(x-5)\right)$$

$$\overset{①}{=} \frac{1}{2}\left(\log 3 + \log x - \log(x-5)\right)$$

$$d) \quad \ln(y(y-1)^2)$$

$$\overset{①}{=} \ln y + \ln(y-1)^2$$

$$\overset{③}{=} \ln y + 2\ln(y-1)$$

WARNING:

$$\ln x^2 = 2\ln x$$

$$\ln x^2 \neq (\ln x)^2$$

③ EXAMPLE

Condense these expressions using properties of logarithms.

$a)\quad \log_5(2x)+\log_5(3y) \overset{①}{=} \log_5(2x\cdot 3y)$

$= \log_5(6xy)$

$b)\quad 5[\ln x-\frac{1}{2}\ln(x+4)] \overset{③}{=} 5(\ln x-\ln(x+4)^{1/2})$

$\overset{②}{=} 5(\ln(\frac{x}{\sqrt{x+4}})) \overset{③}{=} \ln(\frac{x}{\sqrt{x+4}})^5$

$c)\quad 3[\frac{1}{2}\log(x+6)-2\log(x-1)]$

$\overset{③}{=} 3(\log(x+6)^{1/2}-\log(x-1)^2]$

$\overset{②}{=} 3(\log(\frac{\sqrt{x+6}}{(x-1)^2}))$

$\overset{③}{=} \log(\frac{\sqrt{x+6}}{(x-1)^2})^3$