Math 1030 #11b

Linear vs Exponential Growth

The Impact of Doubling

Doubling

Absolute Growth

Relative growth

The power of doubling can be seen in this example:

EX 1: Your rich uncle gives you a dollar and says, "I will double this amount tomorrow and double that amount the next day. I will continue this as long as you do not miss any part of a day of school."

a) How much will you get on the sixth continuous day of attending school?

on 6th day, you get \$32.

n	\$
1	1
2	2
3	$2^2=4$
4	$2^2(2)=2^3=8$
5	$2^4=16$
6	$2^5=32$

b) On what day will he have to give you over a million dollars?

$n=?$ when you get \$1,000,000

$$2^{n-1} \geq 1{,}000{,}000$$

check: $2^{19}=524{,}288$

and $2^{20}=1{,}048{,}576$

when $n-1=20$, you'll get \$1,048,576 that day

$n=21$

$\Rightarrow$ on the 21st day, you'll get over \$1 million

EX 2: Say that a bacteria growing in a lab doubles every 3 minutes. You begin at noon with 2 bacteria in a bottle. In 2 hours, the bottle is full.

$n = \#$ of 3-min. increments

n	# bacteria
0	2^1
1	2^2
2	2^3
3	2^4
⋮	
n	2^{n+1}

a) How many bacteria fit in the bottle?

2 hrs = 40 3-min. increments

$\Rightarrow n = 40$

$\Rightarrow \#$ bacteria $= 2^{40+1} = 2^{41}$

$\simeq 2.199 \times 10^{12}$ bacteria

b) At what time is the bottle half-full?

$n = ?$ when $2^{n+1} = \frac{1}{2}(2^{41})$

$2^{n+1} = 2^{40} \Leftrightarrow n+1 = 40 \Leftrightarrow n = 39$

at time 1:57 p.m.

c) What percent of the bottle is filled at 1:51?

at 1:51, $20 + 17 = 37$ 3-min. increments have passed since noon (since $\frac{51}{3} = 17$)

$\Rightarrow n = 37$, # bacteria $= 2^{37+1} = 2^{38}$

$$\frac{\text{part}}{\text{full}} = \frac{2^{38} \text{ bacteria}}{2^{41} \text{ bacteria}} = \frac{1}{2^3} = \frac{1}{8} = 12.5\%$$

EX 3: Seventy percent of the surface of the earth is covered with water. That leaves about 1.53×10^{14} m^2 of 'land'. If the population in the year 2000 was six billion and the population doubles every fifty years, when will we each have only 1 m^2 of space to occupy? n = # of 50-yr increments

n=0 in year 2000

note: $1.53\times10^{14} \div (6\times10^9) \simeq 25{,}500$

year	n	population	space to occupy (m^2 per person)
2000	0	$6{,}000{,}000{,}000 = 6\times10^9$	25,500
2050	1	$2(6\times10^9) = 12\times10^9$	$25500(\frac{1}{2}) = 12750$
2100	2	$2^2(6\times10^9) = 24\times10^9$	$25500(\frac{1}{2})^2 = 6375$
2150	3	$2^3(6\times10^9) = 48\times10^9$	$25500(\frac{1}{2})^3 = 3187.5$
⋮	⋮ n	⋮ $2^n(6\times10^9)$	⋮ $25500(\frac{1}{2})^n$

n = ? when space to occupy (per person) = 1 m^2

$$25500\left(\tfrac{1}{2}\right)^n = 1$$

$$\left(\tfrac{1}{2}\right)^n = \frac{1}{25{,}500}$$

$$\frac{1}{2^n} = \frac{1}{25{,}500}$$

$$2^n = 25{,}500$$

note: $2^{14} = 16{,}384$ and $2^{15} = 32{,}768$

⇒ we'll each have only 1 m^2 of land to occupy when n is between 14 and 15.

⇒ year is $2000 + 14.5(50) = 2725$

i.e. this happens around the year 2725 (if growth continues to double in this fashion)