Math 1030 #12a

Exponential Decay

Exponential Growth

Doubling Time and Half-Life

Doubling Time

Half-life

Doubling

Exponential growth leads to repeated doublings.

Exponential decay leads to repeated halving.

EX 1: If you get a salary increase of 10% each year, in what year will your salary be double what it is today?

year	salary
0	x
1	$1.1x$
2	$1.1(1.1x) = 1.1^2 x$
3	$1.1(1.1^2 x) = 1.1^3 x$
4	$1.1^4 x = 1.4641x$
5	$1.1^5 x = 1.61051x$
6	$1.1^6 x = 1.771561x$
7	$1.1^7 x = 1.9487171x$

$\Rightarrow$ in year 7, salary is almost doubled

note:

$1.1 = 1 + 0.1$

$= 100\% + 10\%$ more

$\Rightarrow 1.1x$ = my whole salary + 10% additional salary.

After a time, t, an exponentially growing quantity with a doubling time of T_{double} increases in size by a factor of $2^{t/T}$. The new value is related to the initial value by

$$\boxed{\textit{new value} = \textit{initial value} \times 2^{t/T}}$$

$$\boxed{\text{growth factor} = 2^{t/T}}$$

t = time you're interested in

T = doubling time

EX 2: Suppose your bank account has a doubling time of 11 years. By what factor does your balance increase in 34 years?

$T = 11$ yrs, $t = 34$ yrs

growth factor $= 2^{t/T} = 2^{34/11} \approx 8.5203$

$\Rightarrow$ your balance increases by a factor of about 8.5

EX 3: The initial population of a town is 10,000 and it grows with a doubling time of 8 years. What will the population be in

a) 12 years? $T = 8$ yrs

$t = 12$ yrs

b) 24 years? $t = 24$ yrs

new population = initial pop. * $2^{t/T}$

(a) new pop. = $10000\left(2^{12/8}\right) \simeq 28284.27$

$\simeq 28284$

(b) new pop. = $10000\left(2^{24/8}\right) = 10000\left(2^{3}\right)$

$= 80{,}000$

It can be interesting to look at the time it takes your money in a bank to double.

EX 4: If you place $1000 in an account that pays 9% annual interest, compounded annually, during what year will it double?

$$A = P(1+APR)^y$$ (compound interest formula)

y = Year	Amount = A
0	1000
1	$1000(1+0.09)^1 = 1000(1.09)^1 = \$1090$
2	$1000(1.09)^2 = \$1188.10$
3	$1000(1.09)^3 \simeq \$1295.03$
4	$1000(1.09)^4 \simeq \$1411.58$
5	$1000(1.09)^5 \simeq \$1538.62$
6	$1000(1.09)^6 \simeq \$1677.10$
7	$1000(1.09)^7 \simeq \$1828.04$
8	$1000(1.09)^8 \simeq \$1992.56$
9	$1000(1.09)^9 \simeq \$2171.89$

$\Rightarrow$ it took a little over 8 years for your money to double

Rule of 70

For a quantity growing exponentially at a rate of $P\%$ per time period, the doubling time is approximately

$$\boxed{T_{double} \approx \frac{70}{P}}$$ (time periods)

This works best for small growth rates and breaks down for growth rates over about 15%.

EX 5: Determine about how many years it will take you to double your money at these annual interest rates.

a) 3%

$T_d \approx \frac{70}{3}$

≈ 23.3 yrs

b) 5%

$T_d \approx \frac{70}{5}$

$= 14$ yrs

c) 8%

$T_d \approx \frac{70}{8}$

$= 8.75$ yrs

EX 6: The world population was about 6.8 billion in 2005 and was growing at a rate of about 1.2% per year.

a) What is the approximate doubling time?

$T_d \approx \frac{70}{1.2} \simeq 58.3$ yrs

b) If this growth rate continues, what would the population be in 2019?

$t = 14$ $\quad A = P(1+r)^t = 6.8(1+0.012)^{14}$

$\simeq 8.04$ billion people

Exact doubling time formula:

$$\boxed{T_{double} = \frac{log_{10}(2)}{log_{10}(1+r)}}$$ where r is a decimal and positive.

Note: The units of time for r and T must be the same (per month, year, etc.)

EX 7: Oil consumption is increasing at a rate of 2.2% per year.

$r = 0.022$

a) What is the approximate doubling time?

$T_d \approx \frac{70}{2.2} \simeq 31.8$ yrs

b) What is the exact doubling time?

$T_d = \frac{\log(2)}{\log(1+0.022)} \simeq 31.852$ yrs