Math 1030 #12a
Doubling Time and Half-Life
Doubling Time

Exponential growth leads to repeated doublings.

Exponential decay leads to repeated halving.

EX 1:

If you get a salary increase of 10% each year, in what year will your salary be double what it is today?

After a time, t, an exponentially growing quantity with a doubling time of T_{double} increases in size by a factor of $2^{t/T}$. The new value is related to the initial value by

$$\text{new value} = \text{initial value} \times 2^{t/T}.$$

EX 2:

Suppose your bank account has a doubling time of 11 years. By what factor does your balance increase in 34 years?

EX 3:

The initial population of a town is 10,000 and it grows with a doubling time of 8 years. What will the population be in

a) 12 years?

b) 24 years?

It can be interesting to look at the time it takes your money in a bank to double.

EX 4:

If you place $1000 in an account that pays 9% annual interest, compounded annually, during what year will it double?

$$A = P(1 + APR)^y$$

Year	Amount

Rule of 70

For a quantity growing exponentially at a rate of P% per time period, the doubling time is approximately

$$T_{\text{double}} \approx \frac{70}{P}$$

This works best for small growth rates and breaks down for growth rates over about 15%.

EX 5:

Determine about how many years it will take you to double your money at these annual interest rates.

a) 3%

b) 5%

c) 8%

EX 6:

The world population was about 6.8 billion in 2005 and was growing at a rate of about 1.2% per year.

a) What is the approximate doubling time?

b) If this growth rate continues, what would the population be in 2019?

Exact doubling time formula:

$$T_{\text{double}} = \frac{\log_{10}(2)}{\log_{10}(1+r)} \text{ where } r \text{ is a decimal and positive.}$$

Note: The units of time for r and T must be the same (per month, year, etc.)

EX 7:

Oil consumption is increasing at a rate of 2.2% per year.

a) What is the approximate doubling time?

b) What is the exact doubling time?