Math 1030 #12b
Doubling Time and Half-Life
Half-Life

Half-life is the time it takes for half of a population to vanish if it decreases by the same percent each time period.

EX 1:

If a city of 1000 people is decreasing by 10% each year, when will there be half as many people?

Year	Number of people

After a time, t, an exponentially decaying quantity with a half life of T_{half} decreases in size by a factor of $(1/2)^{t/T}$. The new value is related to the initial value by

$$\text{new value} = \text{initial value} \times (1/2)^{t/T}.$$

<u>Approximate Half-life</u>

For a quantity decaying exponentially at a rate of P% per time period $T_{\text{half}} \approx \frac{70}{P}$
This works best for small rates and breaks down for rates over about 15%.

EX 2:

Radioactive carbon-14 has a half-life of about 5700 years. It collects in organisms only while they are alive. Once they are dead, it only decays. What fraction of carbon-14 in an animal bone still remains 800 years after the animal has died?

EX 3:

A clean-up project is reducing the concentration of a pollutant in the water supply, with a 3.5% decrease per week.

a) What is the approximate half-life of the concentration of the pollutant?

b) What fraction of the original will remain after one year?

Exact half-life formula:

$$T_{\text{half}} = -\frac{\log_{10}(2)}{\log_{10}(1+r)}$$ where r is a decimal and negative.

Note: The units of time for r and T must be the same (per month, year, etc.)

EX 4:

Suppose the Russian ruble is falling in value against the dollar at 11% per year.

a) Approximately how long will it take the ruble to lose half its value?

b) Exactly how long will it take the ruble to lose half its value?