Math 1030 #16D

Using Roots to Find Rates

Exponential Decay and Growth:

Decay

$$Q = Q_0(1-r)^t$$

Growth

$$Q = Q_0(1+r)^t$$

Q_0 = initial amount, Q = final amount, r = rate, t = time

Use different techniques to find different parts of the model:

- Use division to find Q_0:

Ex: $700 = \boxed{Q_0}(1-0.03)^8$

$$\frac{700}{(1-0.03)^8} = Q_0$$

$$\boxed{552.586 = Q_0}$$

- Take logs of both sides to find t:

Ex: $700 = 200(1+0.03)^t$

$$\frac{700}{200} = (1+0.03)^t$$

$$3.5 = (1.03)^t$$

$$\log(3.5) = \log\left[(1.03)^{t}\right]$$

$$\log(3.5) = t \cdot \log(1.03)$$

$$\frac{\log(3.5)}{\log(1.03)} = t$$

$$\boxed{42.382 \approx t}$$

- Take roots of both sides to find r:

Ex: $700 = 200(1+\boxed{r})^8$

Example 1: Solve the equations

$2^5 = 32$

a) $x^2 = 16$

$\sqrt[2]{x^2} = \pm\sqrt[2]{16}$

$x = \pm 4$

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b) $x^5 = 32$

$\sqrt[5]{x^5} = \sqrt[5]{32}$

$x = 2$

c) $x^5 = 33$

$\sqrt[5]{x^5} = \sqrt[5]{33}$

$x \approx 2.012$

d) $(x-2)^9 = 2500$

$\sqrt[9]{(x-2)^9} = \sqrt[9]{2500}$

$x - 2 = \sqrt[9]{2500}$

$x = \sqrt[9]{2500} + 2$

$\boxed{x \approx \boxed{4}.385}$

Error Here: Should be 4.385

e) $700 = 200(1 + \boxed{r})^8$

$\frac{700}{200} = (1+r)^8$

$\sqrt[8]{3.5} = \cancel{\pm}\sqrt[8]{(1+r)^8}$

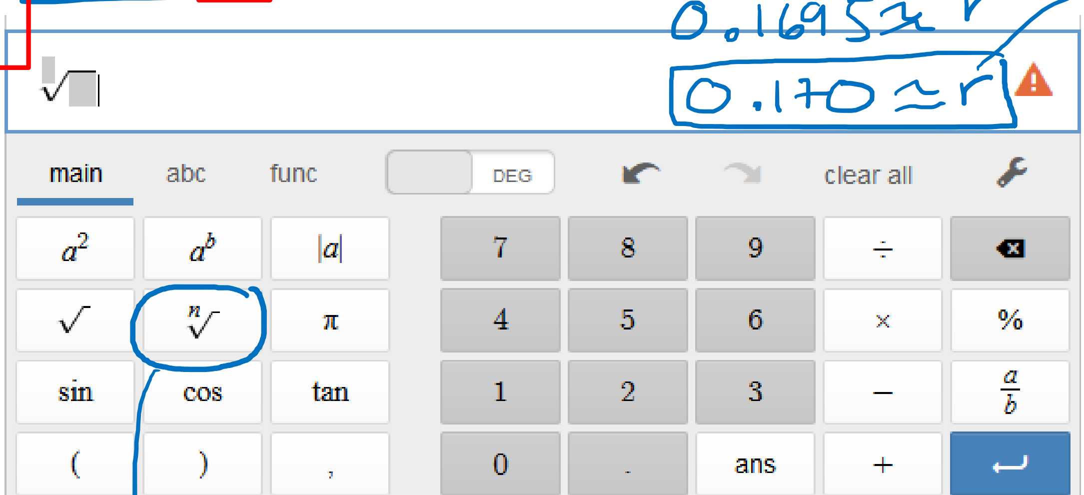
$\sqrt[8]{3.5} = 1 + r$

$\sqrt[8]{3.5} - 1 = r$

$0.1695 \approx r$

$\boxed{0.170 \approx r}$

rate $\approx$ 17%



nth root

$\sqrt[\boxed{5}]{\boxed{33}}$

Roots and Fractional Exponents

- Exponent Properties: $(7^2)(7^3) = (7 \cdot 7)(7 \cdot 7 \cdot 7) = 7^5$

  $(7^2)(7^3) = 7^{2+3} = 7^5$

- Fractional Exponents: $(7^{1/2})(7^{1/2}) = 7^{1/2+1/2} = 7^1 = 7$

- Square Roots: $(\sqrt{7})(\sqrt{7}) = 7$

- Root-Fractional Exponent Connection: $\boxed{\sqrt{7} = 7^{1/2}}$

  $\sqrt[2]{7} = 7^{1/2}$

  $\boxed{\sqrt[n]{x} = x^{1/n}}$

Ex 2: Rewrite the following with rational exponents, then calculate them.

a) $\sqrt[3]{10} = 10^{\frac{1}{3}} \approx 2.154$

b) $\sqrt[4]{81} = 81^{\frac{1}{4}} = 3$ $\qquad 3^4 = 81$

c) $\sqrt[25]{1000} = 1000^{\frac{1}{25}} \approx \underline{1.318}$

$(1.318)^{25} \approx 1000$

Ex 3: In 1990, the population of a city was 20,000. In 2016, the population had grown to 60,000. Find the average annual rate of growth.

$Q_0 = 20{,}000$

$Q = 60{,}000$

$t = 2016 - 1990$
$= 26$

$r = ?$

$Q = Q_0(1+r)^t$

$60{,}000 = 20{,}000(1+r)^{26}$

$\frac{60{,}000}{20{,}000} = (1+r)^{26}$

$\sqrt[26]{3} = \cancel{\pm}\sqrt[26]{(1+r)^{26}}$

$\sqrt[26]{3} = 1+r$

$\sqrt[26]{3} - 1 = r$

$0.0431 \approx r$

$\boxed{\text{4.31\% average annual growth}}$

Ex 4: A drug has a half life in the body of 14 hours. Find the hourly rate of decay.

$Q = Q_0(1-r)^t$

$t = 14$ hours

$Q = \frac{1}{2}Q_0$

$\frac{1}{2}Q_0 = Q_0(1-r)^{14}$

$\frac{\frac{1}{2}\cancel{Q_0}}{\cancel{Q_0}} = \frac{\cancel{Q_0}(1-r)^{14}}{\cancel{Q_0}}$

$\frac{1}{2} = (1-r)^{14}$

$\sqrt[14]{\frac{1}{2}} = \cancel{\pm}\sqrt[14]{(1-r)^{14}}$

$\sqrt[14]{0.5} = 1-r$

$r = 1 - \sqrt[14]{0.5}$

$r = 0.0483$

hourly decay rate of 4.83%

rate -4.83%