

Math 1030 #16D

Using Roots to Find Rates

Exponential Decay and Growth:

$$Q = Q_0(1 - r)^t$$

$$Q = Q_0(1 + r)^t$$

Q_0 = initial amount, Q = final amount, r = rate, t = time

Use different techniques to find different parts of the model:

- Use division to find Q_0 :

$$\text{Ex: } 700 = Q_0(1 - 0.03)^8$$

- Take logs of both sides to find t :

$$\text{Ex: } 700 = 200(1 + 0.03)^t$$

- Take roots of both sides to find r :

$$\text{Ex: } 700 = 200(1 + r)^8$$

Example 1: Solve the equations

a) $x^2 = 16$

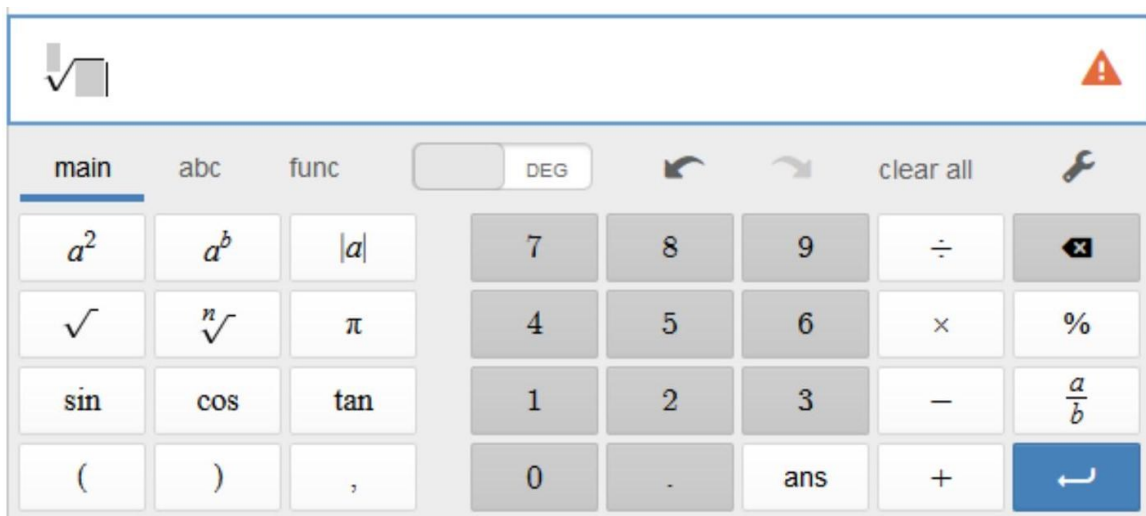
b) $x^5 = 32$

c) $x^5 = 33$

d) $(x - 2)^9 = 2500$

e) $700 = 200(1 + r)^8$

Look for calculation error in video in last step of 1d. It is corrected in post notes.



Roots and Fractional Exponents

- Exponent Properties: $(7^2)(7^3) = (7 \cdot 7)(7 \cdot 7 \cdot 7) = 7^5$

$$(7^2)(7^3) = 7^{2+3} = 7^5$$

- Fractional Exponents: $(7^{1/2})(7^{1/2}) = 7^{1/2+1/2} = 7$

- Square Roots: $(\sqrt{7})(\sqrt{7}) = 7$

- Root-Fractional Exponent Connection:

$$\sqrt{7} = 7^{1/2}$$

$$\sqrt[2]{7} = 7^{1/2}$$

$$\sqrt[n]{x} = x^{1/n}$$

Ex 2:

Rewrite the following with rational exponents, then calculate them.

a) $\sqrt[3]{10}$

b) $\sqrt[4]{81}$

c) $\sqrt[25]{1000}$

Ex 3:

In 1990, the population of a city was 20,000. In 2016, the population had grown to 60,000. Find the average annual rate of growth.

Ex 4:

A drug has a half life in the body of 14 hours. Find the hourly rate of decay.