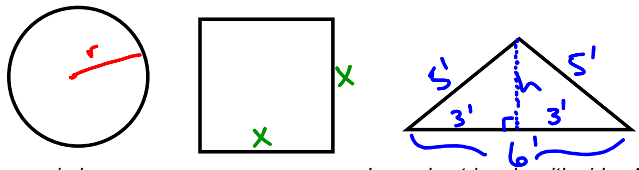
Math 1030 #18d

Problem Solving in Geometry

Optimization

Optimization problems seek "best solutions" to various problems.

Ex 1) If each of these shapes has a perimeter of 16 ft, put them in order of area from least to greatest.



circle   square   *Isosceles triangle with sides 5', 5' 6'*

circle

$P = 16\,ft = 2\pi r$

$r = \frac{16}{2\pi}\,ft = \frac{8}{\pi}\,ft$

$A_c = \pi r^2 = \pi\left(\frac{8}{\pi}\,ft\right)^2 = \pi\left(\frac{64}{\pi^2}\right)ft^2 = \frac{64}{\pi}\,ft^2$

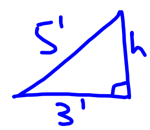
$\simeq 20.4\,ft^2$

Square

$P = 4x = 16\,ft$

$\Rightarrow x = \frac{16}{4}\,ft = 4\,ft$

$A_s = x^2 = (4\,ft)^2 = 16\,ft^2$



triangle

$A_T = \frac{1}{2}bh$

$= \frac{1}{2}(6\,ft)(4\,ft)$

$= 12\,ft^2$

use Pythagorean Thm to find h:

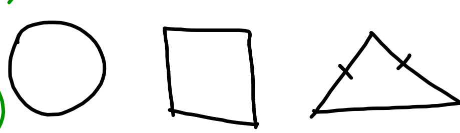
$3^2 + h^2 = 5^2$

$h^2 = 25 - 9$

$h^2 = 16 \Rightarrow h = 4\,ft$

from greatest area (on left) to smallest area (on right)

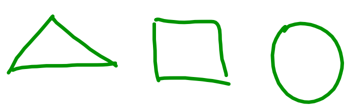
$A_c > A_s > A_T$



where each shape has same perimeter

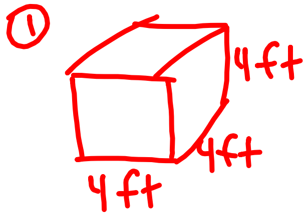
from least to greatest

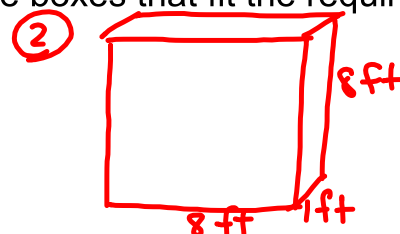
$A_T < A_s < A_c$

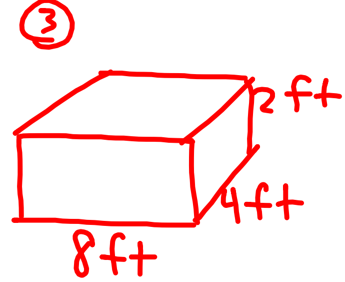


EX 2: You are to design a rectangular box with a volume of 64 ft^3.

a) Draw three sample boxes that fit the requirements.







b) Determine the surface area of of each.

① $S_1 = 6(4 \cdot 4)$
$= 96 \text{ ft}^2$

② $S_2 = 2(8 \cdot 8) + 2(8 \cdot 1) + 2(8 \cdot 1)$
$= 160 \text{ ft}^2$

③ $S_3 = 2(8 \cdot 2) + 2(8 \cdot 4) + 2(2 \cdot 4)$
$= 112 \text{ ft}^2$

c) State the price of each if the materials cost \$3.00/ft^2.

① $\text{Cost} = \frac{\$3.00}{ft^2}(96 ft^2)$
$= \$288$

② $\text{Cost} = 3.00(160)$
$= \$480$

③ $\text{Cost} = 3.00(112)$
$= \$336$