

REVIEW

Powers
and
Roots

Basics of Powers

2^5 means = 2 multiplied by itself 5 times

2^0 means = 1
(1. no twos)

2^{-5} means = $\frac{1}{2^5} = \frac{1}{32}$
1 divided by (five twos multiplied together)

EX 1: Evaluate these.

a) $4^3 = 4 \cdot 4 \cdot 4 = 64$

b) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

c) $8^0 = 1$

d) $9^1 = 9$

$$\begin{aligned} 2^5 &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32 \\ 2^4 &= 2 \cdot 2 \cdot 2 \cdot 2 = 16 \quad \swarrow \div 2 \\ 2^3 &= 8 \quad \swarrow \div 2 \\ 2^2 &= 4 \quad \swarrow \div 2 \\ 2^1 &= 2 \quad \swarrow \div 2 \\ 2^0 &= 1 \quad \swarrow \div 2 \\ 2^{-1} &= \frac{1}{2} \quad \swarrow \div 2 \\ 2^{-2} &= \frac{1}{4} \quad \swarrow \div 2 \\ 2^{-3} &= \frac{1}{8} \quad \swarrow \div 2 \end{aligned}$$

Power Rules

When multiplying powers of a like base, *add* the exponents.

$$2^3 2^5 = \underbrace{2 \cdot 2 \cdot 2}_{3 \text{ factors}} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{5 \text{ factors}} = 2^8$$

When dividing powers of a like base, *subtract* the exponents.

$$\frac{2^7}{2^5} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = 2^2 \qquad \frac{2^7}{2^5} = 2^{7-5} = 2^2$$

When a power is raised to another power, *multiply* the exponents.

$$(2^3)^4 = (\underbrace{2 \cdot 2 \cdot 2}_{3 \text{ factors}})(\underbrace{2 \cdot 2 \cdot 2}_{3 \text{ factors}})(\underbrace{2 \cdot 2 \cdot 2}_{3 \text{ factors}})(\underbrace{2 \cdot 2 \cdot 2}_{3 \text{ factors}}) \qquad (2^3)^4 = 2^{3 \cdot 4} = 2^{12}$$

EX 2: Evaluate these.

a) $5^3 5^4 = 5^{3+4} = 5^7$

b) $(3^2)^4 = 3^{2 \cdot 4} = 3^8$

c) $\frac{4^7}{4^5} = 4^{7-5} = 4^2 = \boxed{16}$

d) $\frac{3^4}{3^7} = 3^{4-7} = 3^{-3} = \boxed{\frac{1}{3^3}}$

EX 3: Evaluate these.

$$\text{a) } \frac{3^2 \cdot 2^4}{2^5 \cdot 3^3} = 2^{4-5} \cdot 3^{2-3} \\ = 2^{-1} \cdot 3^{-1} = \frac{1}{2} \cdot \frac{1}{3}$$

$$\text{b) } \frac{(3^2)^3}{(2^3)^4} = \frac{3^6}{2^{12}} = \frac{1}{6}$$

$$\text{c) } \frac{3^3 \cdot 2^5}{2^4 \cdot 3^2} = 3^{3-2} 2^{5-4} = 3^1 2^1 = 3 \cdot 2 = 6$$

$$\text{d) } \frac{(3^3 \cdot 2^4)^2}{(2^5 \cdot 3^2)^3} = \frac{(3^3)^2 (2^4)^2}{(2^5)^3 (3^2)^3} = \frac{3^6 2^8}{2^{15} 3^6} \\ = 3^{6-6} 2^{8-15} = 3^0 2^{-7} = \frac{1}{2^7}$$

Power Rules

$$b^m \cdot b^n = b^{m+n}$$

$$\frac{b^m}{b^n} = b^{m-n}$$

$$(b^m)^n = b^{m \cdot n}$$

Basics of Roots

$$\sqrt{81} = 9 \text{ because } 9^2 = 81$$

$$\sqrt[3]{64} = 4 \text{ because } 4^3 = 64$$

$$\sqrt[4]{16} = 2 \text{ because } 2^4 = 16$$

$$\sqrt[5]{243} = 3 \text{ because } 3^5 = 243$$

EX 4: Evaluate these.

$$\text{a) } \sqrt[5]{32} = 2 \text{ because } 2^5 = 32$$

$$\text{b) } \sqrt[4]{81} = 3 \text{ because } 3^4 = 81$$

$$\text{c) } \sqrt[3]{125} = 5 \text{ " } 5^3 = 125$$

$$\text{d) } \sqrt{10,000} = 100 \text{ because } 100^2 = 10,000$$