2.2 Polynomial Functions of Higher Degree

- Use transformations to sketch graphs of polynomial functions
- Determine end behavior by looking at the leading coefficient
- Find and use zeros of polynomial functions as sketching aids

Here are some graphs of polynomial functions:

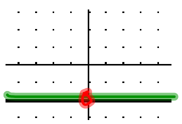
Zeros

End Behavior

Leading coefficient

y-intercept

Constant function



no zeros

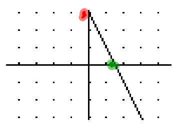
$y = -2$

$y=-2$

$(0,-2)$

$y = mx + b$

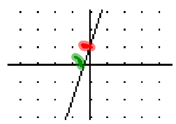
Linear function



$(1.5, 0)$

$a<0$

$(0,3)$



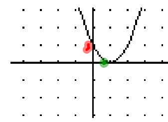
$(-.3, 0)$

$a>0$

$(0,1)$

$y = ax^2 + bx + c$

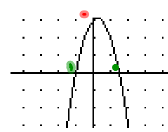
Quadratic function



$(1,0)$ double rt

$a>0$

$(0,1)$



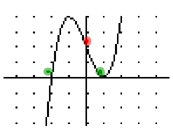
$(-1,0)(1.5,0)$

$a<0$

$(0,3)$

$y = ax^3 + bx^2 + cx + d$

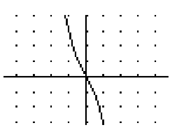
Cubic function



$(-2,0)(1,0)$ double

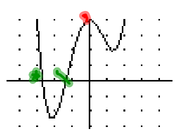
$a>0$

$(0,2)$



$a<0$

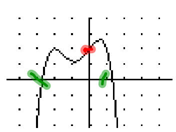
Quartic function



2 roots

$a>0$

$(0,4)$

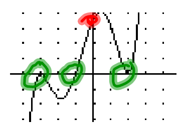


2 roots

$a<0$

$(0,2)$

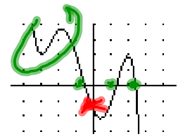
Other polynomial functions



2 double rts

1 single rt

$a>0$



3 roots

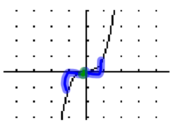
$a<0$

$(0,-1)$

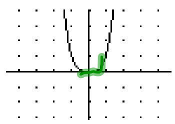
Transformations of higher degree polynomial functions.

$y = a(x-h)^n + k$

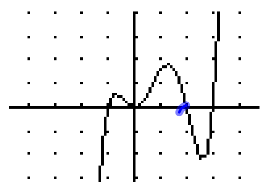
If this is $y = x^3$



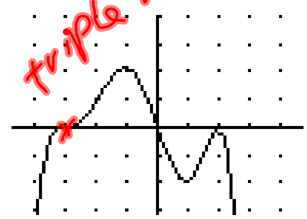
If this is $y = x^4$



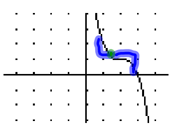
Guess at these:



triple rt



then what is this?



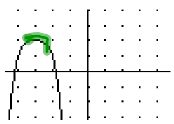
$a<0$

$a=-1$

rt 2 up 1

$y=-1(x-2)^3+1$

then what is this?



$a=-1$

left 3

up 2

4

$y=-(x+3)^4+2$

$y=a(x+1)(x)^2(x-2)(x-3)$

$a>0$

$y=(x-3)^3(x)(x-2)^2(a)$

6th degree

$a<0$

Sketching graphs of polynomial functions

If the polynomial factors:
- factor it ✓
- place roots ✓
- y-intercept ✓ $x=0$ $y:0$
- end behavior

$a>0$ ↖ ↗

$$f(x) = x^4 - x^3 - 20x^2$$

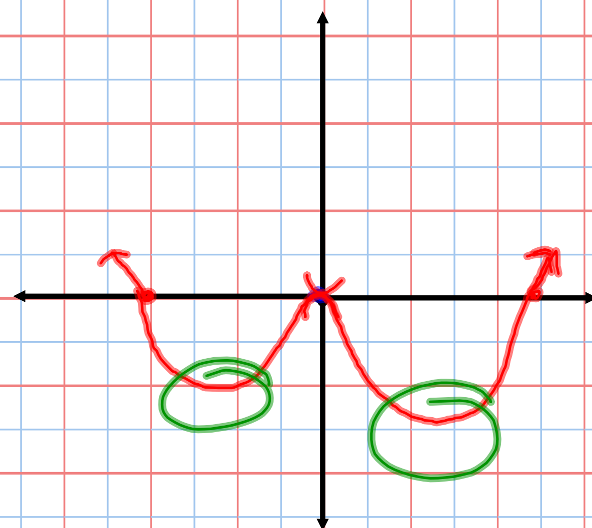
$= x^2(x^2 - x - 20)$

$= x^2(x-5)(x+4)$

$x = 0$ double rt

$x = 5$

$x = -4$



$$f(x) = x^3 - 3x + 1$$

If it does not factor:
- end behavior $a>0$, ↙ ↗
- y- intercept — $(0,1)$
- estimate some points

$x = -2$ $y = (-2)^3 - 3(-2) + 1$
$-8+6+1 = -1$

$x = -1$ $y = (-1)^3 - 3(-1) + 1$
$-1+3+1 = 3$

$x = 0$ $y = 0^3 - 30 + 1 = 1$

$x = 1$ $y = 1^3 - 3(1) + 1 = -1$
$1-3+1$

$x = 2$ $y = 2^3 - 3(2) + 1$
$8-6+1$
3