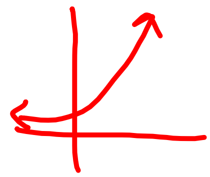
Chapter 3: Exponential and Logarithmic Functions

In section 3.1 you will learn to:

- Recognize, evaluate and graph exponential functions with whole number bases.
- Use exponential functions to determine simple and compound interest.
- Recognize, evaluate and graph exponential functions with base *e*.

$f(x) = 2^x$ <u>ex</u> \$5000 @ 4%

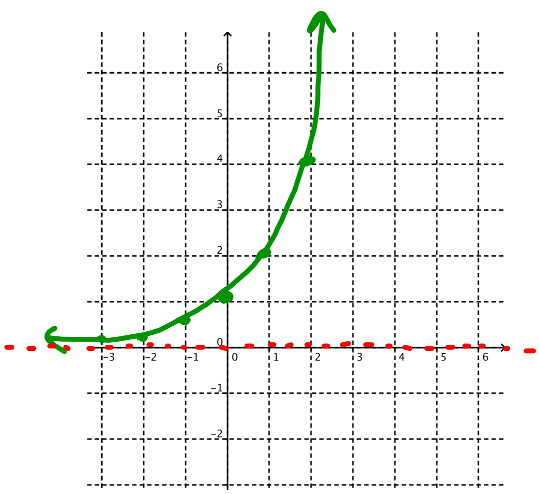
$e \approx 2.718$



An exponential function is one where the independent variable (x) is an exponent.

(note: $y = x^2$ is not an exponential fn)

$f(x) = 2^x$



$f(x) = 2^x$

x	2^x
0	1
1	2
2	4
-1	½
-2	¼
-3	⅛
3	8

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{2^2}$

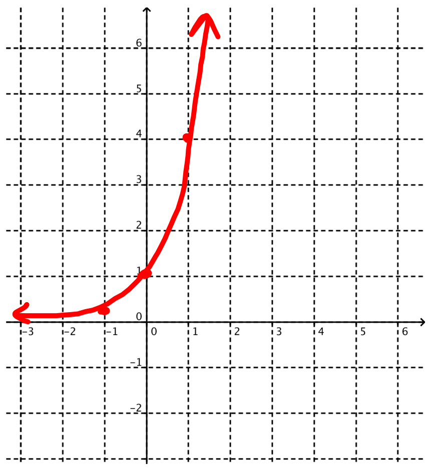
$2^{-3} = \frac{1}{2^3}$

Asymptote: (no VA) $y = 0$ is HA.

Domain of $f(x)$: $x \in \mathbb{R}$ (or $(-\infty, \infty)$)

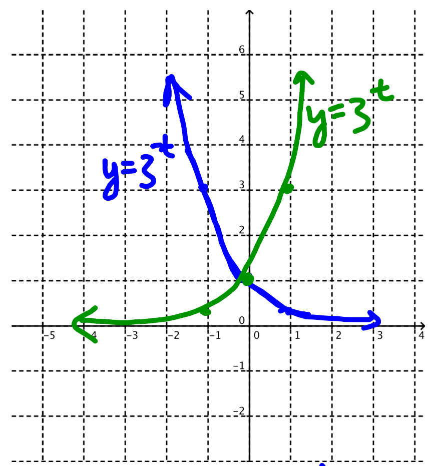
Range of $f(x)$: $y > 0$ (or $(0, \infty)$)

$f(x) = 4^x$



$f(t) = 3^{-t} = (1/3)^t$

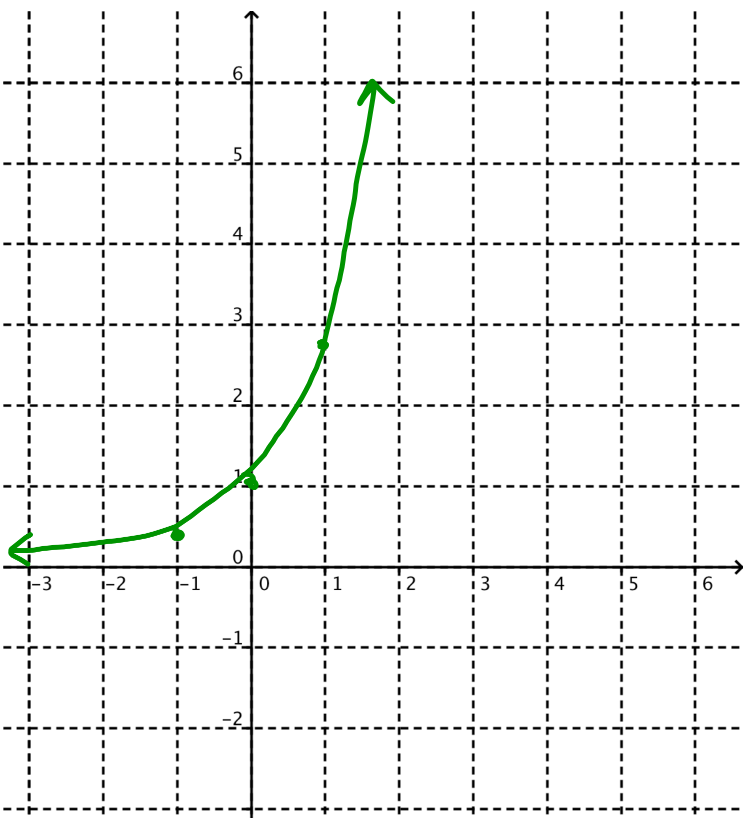
$y = 3^t$



x	y
0	1
1	$1/3$
-1	$\left(\frac{1}{3}\right)^{-1} = 3$

(a base that's less than 1 has same shape as an exponential curve w/ base bigger than 1, except it's reflected across y-axis)

$f(x) = e^x$ where $e \approx 2.718$



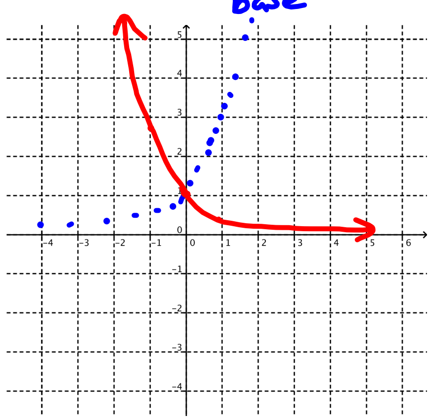
x	y
0	1
1	e
−1	$e^{-1} = \frac{1}{e}$

HA: $y = 0$

Transformations of $\boxed{f(x) = e^x}$

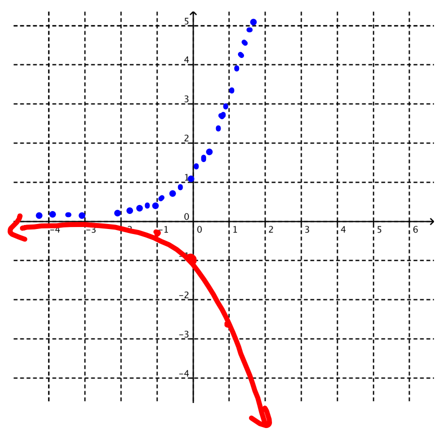
horiz. reflections vert.

$f(x) = e^{-x}$ base



HA: $y=0$

$f(x) = -e^x$

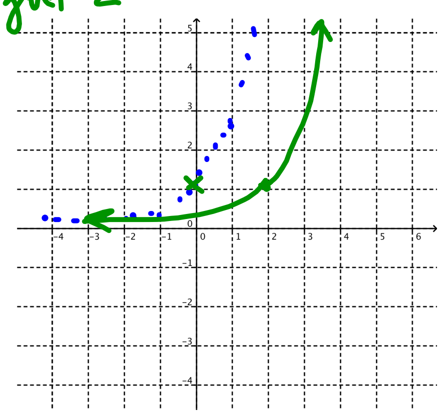


shift right 2

HA: $y=0$

$f(x) = e^{x-2}$

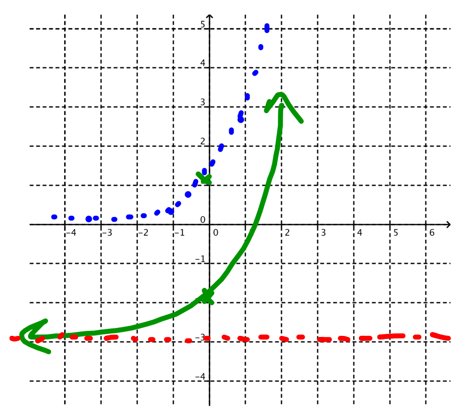
shifts



$f(x) = e^x - 3$

shift down by 3

HA: $y=-3$



Problem 1: If P dollars are invested in an account that pays an interest rate of r (expressed as a percent) compounded annually, how much is in the account after:

a year? $A_1 = \underbrace{P}_{\text{principal}} + \overbrace{Pr}^{\text{interest}} = P(1+r)$

2 years? $A_2 = P(1+r) + [P(1+r)]r = P(1+r)(1+r) = P(1+r)^2$

3 years? $A_3 = P(1+r)^2 + (P(1+r)^2)r = P(1+r)^2(1+r) = P(1+r)^3$

t years? $A_t = P(1+r)^t$

What if we compound it twice year? $A = P\left(1+\frac{r}{2}\right)^{2t}$

Quarterly? $A = P\left(1+\frac{r}{4}\right)^{4t}$

Daily?
$A(t) = A = P\left(1+\frac{r}{365}\right)^{365t}$

Problem 2: As compounding periods become smaller, the compounding can be considered to be instantaneous. This is known as continuous compounding.

The formula for continuous compounding is:

$$\boxed{A_t = Pe^{rt}}$$

P = principal, r = annual interest rate

t = # years.

Discrete compounding: $A_t = P(1+\frac{r}{n})^{nt}$

n = number of compounding times per year.
t = number of years
r = interest rate
P = amount invested
A = amount after t years.

Continuous compounding: $A_t = Pe^{rt}$

Problem 3. $12,000 is invested in an account that pays 4.8 % interest. (annual)
How much can we expect to be in the account after five years if...

$P = 12000$, $r = 0.048$ $t = 5$

discrete

... Interest is compounded annually?

$n = 1$

$A = P(1+\frac{r}{n})^{nt}$

$A = 12000(1+\frac{0.048}{1})^{1(5)} \simeq$ \$15,170.07

... Interest is compounded monthly?

$n = 12$

$A = 12000(1+\frac{0.048}{12})^{12(5)} \simeq$ \$15,247.69

...Interest is compounded daily?

$n = 365$

$A = 12000(1+\frac{0.048}{365})^{365(5)} \simeq$ \$15,254.75

...Interest is compounded continuously?

$A = Pe^{rt} = 12000e^{0.048(5)} \simeq$ \$15,254.99

An interesting question which we will be able to solve later in this chapter is how long it takes to double your investment. A loose estimate can be obtained by the Rule of 72.

$\frac{72}{4.8} \approx 15$ yrs