Systems of Inequalities

In section 7.5 you will learn to:

- Sketch the graphs of inequalities in two variables.
- Solve systems of linear inequalities in two variables.
- Model and solve real-life problems with systems of inequalities in two variables.

Systems of Inequalities

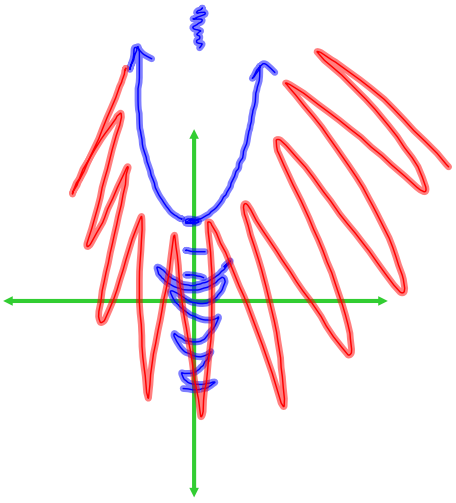
Any curve cuts the plane into two parts. An inequality in 2 variables means we want to shade all points that are solutions of the inequality.

Example 1

Graph solutions to: $y \leq x^2 + 3$

$y = x^2 + 3$

$y \leq x^2 + 3$



If you have two or more inequalities that you wish to solve simultaneously, you need to:

① graph both eqns (curves) on same plane
② shade in region that satisfies all ineq.
③ label vertices (pts of intersection)

Of interest are the vertices (where two equations meet) and the common shaded region.

Example 2

Solve and label vertices for:

① $4x - 6y > -12$
② $x - y > 1$
③ $y > 0$

$-y > 1 - x \Leftrightarrow y < x - 1$

$-6y > -12 - 4x$

$y < \frac{-12}{-6} - \frac{4x}{-6}$

$y < \frac{2}{3}x + 2$

test pt: (0,1)

① $0 - 6 > -12$, $-6 > -12$ true
② $0 - 0 > 1$, $0 > 1$ false
③ $1 > 0$ true

A int ③ & ②
B int ① & ②

A: ③ & ②

$y = 0$, $y = x - 1$

$0 = x - 1$

$x = 1$

(1, 0)

B: ① & ②

$y = \frac{2}{3}x + 2$, $y = x - 1$

$\frac{2}{3}x + 2 = x - 1$

$-\frac{2}{3}x \quad -\frac{2}{3}x \quad +1$

$3 = \frac{1}{3}x$

$9 = x \Rightarrow y = 9 - 1 = 8$

(9, 8)

Example 3

Solve and label vertices

① $y \leq 2x - x^2$
② $0 \leq x + y \Rightarrow -x \leq y \Leftrightarrow y \geq -x$

① $y = -x^2 + 2x$

$y = -x(x - 2) = 0$

$x = 0$, $x - 2 = 0$, $x = 2$

vertex: $x = 1$, $y = -1 + 2 = 1$

test pt: (1, 0)

① $0 \leq 2 - 1 \Leftrightarrow 0 \leq 1$ true
② $0 \leq 1 + 0$ true

B, A: int. pts ① & ②

A & B: $y = -x$, $y = -x^2 + 2x$

$-x^2 + 2x = -x$

$+x \quad +x$

$-x^2 + 3x = 0$

$-x(x - 3) = 0$

$-x = 0$ or $x - 3 = 0$

$x = 0$ $x = 3$

$x = 0$, $y = 0$ → (0, 0)

$x = 3$, $y = -3$ → (3, −3)

Example 4

For a concert event, there are $30 reserved seat tickets, and $20 general admission tickets. There are 2000 reserved sets available and the fire regulations limit the number of paid ticket holders to 3000. The promoter must take in $75,000 in ticket sales. Find and graph the system of inequalities describing the different number of tickets that can be sold.

$x = \#$ \$30 tix

$y = \#$ \$20 tix

(1) $x \leq 2000$

(2) $x + y \leq 3000$

(3) $30x + 20y \geq 75000$

total revenue (\$)

(4) $x \geq 0, y \geq 0$