# Linear Programming

In section 7.6 you will learn to:

- Set up, sketch and solve linear programming problems.
- Use these problems to optimize some quantity.

## Linear Programming

We want to optimize (either maximize or minimize) a function given a set of constraints (inequalities) that must be satisfied (which makes the feasible region.)

Strategy:

1. Sketch the graph of the inequalities (constraints) and shade the feasible region.
2. Find the vertices of that region. (intersection pts)
3. Test all vertices in the objective function to see which produces a maximum or minimum. (equation)

Example 1:

objective fn

Find the maximum value of $z = 6x + 5y$ subject to these constraints:

① $3x + 2y \le 16$ (0, 8) (16/3, 0)

$3x = 16$

$x = 16/3$

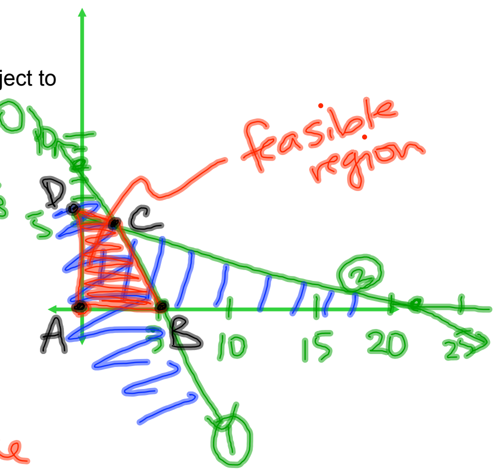
② $x + 4y \le 22$

→ $x \ge 0,\ y \ge 0$

② $(0, 11/2)$ $(22, 0)$

$4y = 22$

$y = 11/2$




test pt

(0,0)

① $0 \le 16$ true

② $0 \le 22$ true

vertices

A: (0, 0)

B: (16/3, 0)

C: (2, 5)

D: (0, 11/2)

pt C int ① + ②

$-2(3x + 2y = 16)$

$x + 4y = 22$

$-6x - 4y = -32$

$+\ \ x + 4y = 22$

$-5x = -10$

$x = 2$

$\Rightarrow x + 4y = 22$

$2 + 4y = 22$

$4y = 20$

$y = 5$

Objective fn

$z = 6x + 5y$

A: $z = 6(0) + 5(0) = 0$

B: $z = 6(16/3) + 5(0) = 32$

C: $z = 6(2) + 5(5) = 37$

D: $z = 6(0) + 5(11/2) = 55/2 = 27\frac{1}{2}$

pt (2, 5) gives max $z = 37$

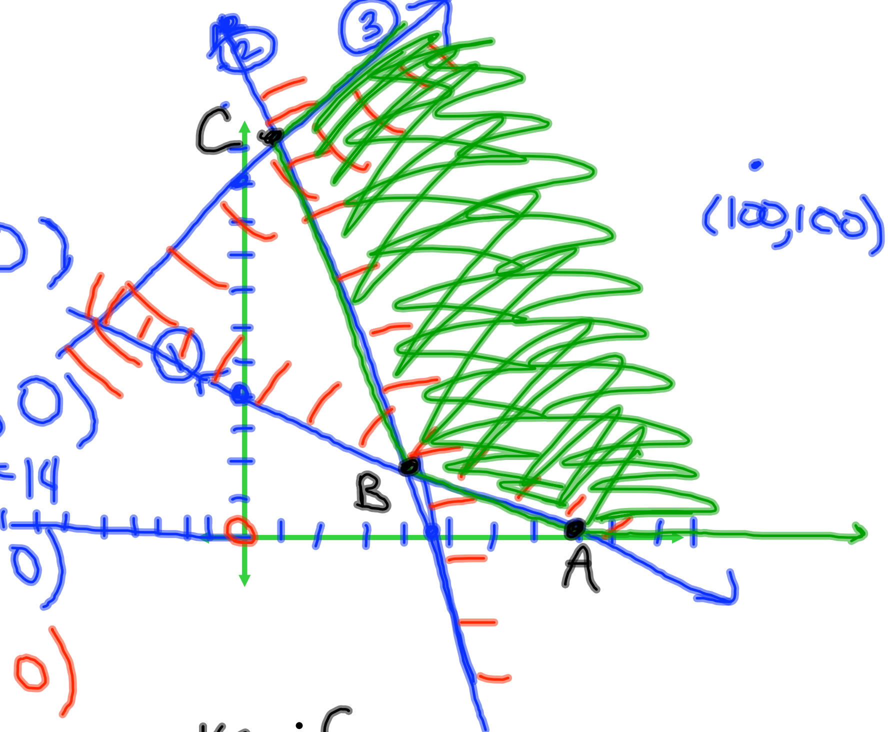
Example 2

obj. fn.

Find the maximum value and where it occurs for $z = 2x + 5y$ subject to

constraints: $x \geq 0$, $y \geq 0$

1. $x + 2y \geq 8$
2. $3x + y \geq 14$
3. $-x + y \leq 10$




① $(0, 4)$ $(8, 0)$
$2y = 8$

② $(0, 14)$ $(14/3, 0)$
$3x = 14$

③ $(0, 10)$ $(-10, 0)$

test pt $(0, 0)$

① $0 \geq 8$ false
② $0 \geq 14$ false
③ $0 \leq 10$ true

verify

A: $(8, 0)$ ✓
B: $(4, 2)$ (int pt ① + ②)
C: $(1, 11)$ (int pt ② + ③)

max. $z = 2x + 5y$

A: $(8, 0)$ $z = 2(8) + 0 = 16$
B: $(4, 2)$ $z = 2(4) + 5(2) = 18$
C: $(1, 11)$ $z = 2(1) + 5(11) = 2 + 55 = 57$

$\Rightarrow$ no max!

min. $z = 16$ at $(8, 0)$

## Example 3

obj. fn

A fruit grower has 150 acres of land available to raise two crops, A and B. It takes 1 day to trim an acre of crop A and two days to trim an acre of crop B, with 240 days per year available for trimming. It takes 0.3 days to pick an acre of Crop A and 0.1 day to pick an acre of crop B with 30 picking days available. The profit is \$140 per acre for crop A and \$235 per acre for crop B. What is the optimal acreage for each fruit? What is the maximum profit?

profit $z = 140x + 235y$

$x$ = # acres crop A

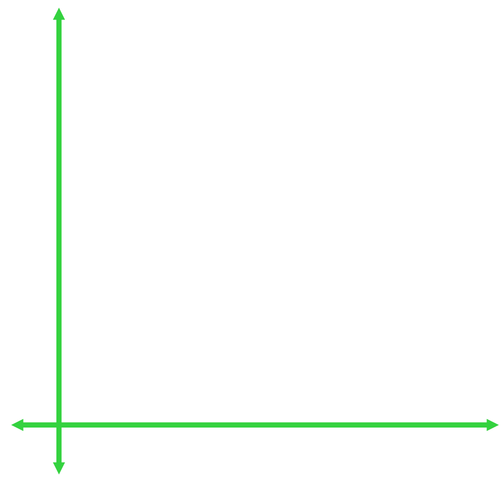
$y$ = # acres crop B

$x \geq 0, y \geq 0$

① $x + y \leq 150$

② $x + 2y \leq 240$

③ $0.3x + 0.1y \leq 30$



(note: $10(0.3x + 0.1y) \leq 30(10)$

use ③ $3x + y \leq 300$ )