9.3 Geometric Sequences and Series

In sections 9.3 you will learn to:

- Recognize, write and find the nth terms of geometric sequences.
- Find the nth partial sums of geometric sequences.
- Find the sums of infinite geometric sequences.
- Use geometric sequences to model and solve real-life problems.

A sequence $a_1, a_2, a_3, \ldots, a_n$ is said to be geometric is the ratio between consecutive terms remains constant.

Which of these are geometric sequences?

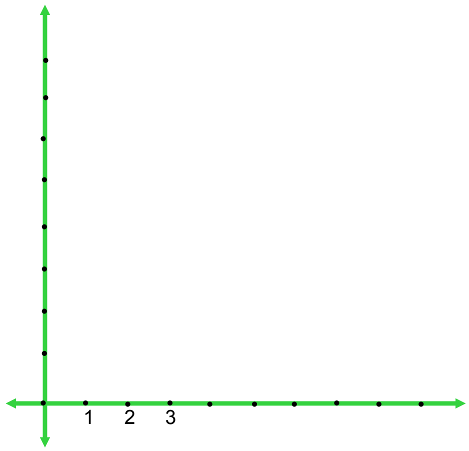
3, 1, 1/3, 1/9, ... 2, 4, 8, 16, ...

1, 4, 9, 16, ... 3, 6, 12, 24, 48, ...

Example 1: Suppose $a_3=4$ and $a_7=1/4$ in a geometric sequence. Find the first seven terms of the sequence.

Example 2:

How would you describe the graph of a geometric sequence?



Example 3:

Suppose a ball is dropped from a height of 9 feet. The elasticity of the ball is such that it bounces up two-thirds of the distance that it has fallen. If this elasticity property remains in effect, how high will the ball bounce after hitting the ground ten times?

A *finite geometric series* is the sum S_n of the first n terms of a finite geometric sequence.

$$Sn = a_1 + (a_1 r) + (a_1 r^2) + (a_1 r^3) + \ldots + (a_1 r^{(n-1)})$$

S_n can be found by computing $S_n = \dfrac{a(1-r^n)}{1-r}$.

Example 4:

Find a formula for the n^{th} partial sum of the geometric series 3 + 6 + 12 + ...
Use the formula to compute S_6.

Example 5:

a) Use the summation notation to write this series, determine a formula for the nth partial sum and find the sixth partial sum using the formula:

$$S_n = \frac{a(1-r^n)}{1-r}$$

a) 1 + 0.7 + 0.49 + 0.343 + ...

b) $\sum_{j=1}^{10} 2(0.1)^j =$

There is a mistake in Problem 5b. See if you can spot it when watching the video. The mistake is corrected in the completed lecture notes.

If the common ratio is between -1 and 1 ($|r| < 1$) in an infinite geometric series, the sum will converge to a finite sum. This is because r^n approaches zero as n increases without bound.

The formula for an infinite sum is:

$$S_\infty = \sum_{j=1}^{\infty} a\,r^j = \frac{a}{1-r}$$

Where a is the first term, a_1 and $|r| < 1$

Example 6:

Compute the infinite sum of the two previous examples:

a) 1 + 0.7 + 0.49 + 0.343 + ...

b) $\sum_{j=1}^{\infty} 2(0.1)^j =$

Example 7:

In the example of the bouncing ball dropped from a height of 9 feet and bouncing up two-thirds of the previous distance on each bounce, what is the total distance it has traveled after bouncing ten times?

Example 8:

In the last two lessons, you decided to save for your trip to Europe. You opened a savings account with $1.00 and on each subsequent day, you deposited a dollar more than on the previous day.

Now you get really brave and each day you deposit twice the amount you did on the previous day, starting with $1.00 on day 1. How much will you deposit on the 30th day? What is the total amount in the account on day 30?