



## Math 1050 ~ College Algebra

### 20 Applications of Exponentials and Logarithms

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1 - z^{n+1}}{1 - z}$$

#### Learning Objectives

- Use compound interest formulas to solve financial applications.
- Solve applications of uninhibited growth and decay.
- Solve additional applications represented by exponential and logarithmic models.

### Simple, Compound and Continuously Compounded Interest

Ex 1: If you invest \$100 at a yearly interest rate of 5%, show how it will grow during the first five years.

Simple Interest

Compound Interest

Ex 2: How much must you invest at age 40 so that you will have a million dollars by the time you retire at 70? Assume an interest rate of 7% compounded annually.

In general, the formula for compound interest is  $A = P \left( 1 + \frac{r}{n} \right)^{nt}$

$A$  = balance after  $t$  years

$P$  = principal

$r$  = annual interest rate

$t$  = number of years

$n$  = number of times it is compounded per year

Ex 3: Show the difference between compounding one time per year and twelve times per year when investing \$1000 at 5% interest for 10 years.

Ex 4: What if the compounding on example 3 is continuous?

### **Exponential Growth and Decay**

$A(t) = A_0 e^{kt}$  is the general formula for the exponential growth or decay of a substance.

Ex 5: The Half-life of radium ( $^{226}\text{R}$ ) is 1620 years. What percent of the radium will still be present after 150 years?

Ex 6: A certain strain of dangerous bacteria is known to grow from 1000 to 5000 in 5 hours. Assume it grows according to the formula above.

a) Determine  $k$ , the growth constant and find a formula for the growth,  $A(t)$ .

b) When will the number present be 12,000?

Ex 7: A car that is priced at \$25,000 new, is worth \$15,000 after two years.

a) Find the linear model of depreciation.  $V = mt + b$

b) Find the exponential model of depreciation.  $V = ae^{kt}$

c) Sketch a graph of the two models.

d) Determine the value of the car at the end of five years for each of the models.

Value in \$1000

