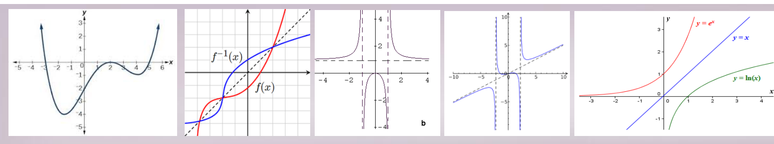

Math 1050 ~ College Algebra

23 Systems of Linear Equations: Augmented Matrices

$$-3x+4y=5$$
$$2x-y=-10$$
$$\begin{bmatrix}-3 & 4\\ 2 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ -10\end{bmatrix}$$

$$\sum_{k=1}^{m} k=\frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k=\frac{1-z^{n+1}}{1-z}$$

Learning Objectives

- Write a system of linear equations as an augmented matrix.
- Perform row operations on a matrix.
- Convert an augmented matrix to row echelon form.
- Convert an augmented matrix to reduced row echelon form.
- Use matrix row operations to solve systems of linear equations.

Solving three equations in three variables takes a bit of writing. We can reduce some of that by writing the set of equations as an **augmented matrix**. A matrix is a structured array of numbers grouped by square brackets. We only write the coefficients and constants.

Ex 1a: Write this set of equations as an augmented matrix.

$$\begin{aligned} x-y+z &= 4\\ x+3y-2z &= -3\\ 3x+2y+2z &= 6 \end{aligned}$$

We can operate on this matrix using the same set of rules that we used for Gaussian elimination in the last section.

1. Exchange two rows.
2. Multiply a row by a nonzero constant.
3. Temporarily multiply a row by a nonzero constant and add it to another row, replacing either of those rows with the result.

Ex 1b: Solve this system of equations from 1a by using these options.

Row Echelon Form

Ex 2: Solve for x, y and z in these augmented matrices representing a set of equations.

a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$

b) $\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$

These are examples of row echelon form and reduced row echelon form. Our goal is to reduce a set of equations to one of these forms, using Gaussian elimination.

Ex 3: Solve this set of equations by reducing them to each of the forms above.

$$\begin{aligned} 2x - 2y + z &= -9 \\ x + y + 2z &= -5 \\ x \quad\;\; - z &= 11 \end{aligned}$$

Ex 4: Set up this set of equations as an augmented matrix and put in reduced row echelon form.

$A = 2$

$B = -1$

$3A + 2C = 0$

$2B - D = -5$

What happens if the system has infinite solutions or no solutions?

Ex 5: Write the solution to each system below.

a) $\begin{bmatrix} 1 & 0 & 0 & \vdots & -3 \\ 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 1 & \vdots & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & \vdots & -3 \\ 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & \vdots & -3 \\ 0 & 1 & 0 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$