

Math 1050 ~ College Algebra

3 Transformations of Functions

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1-z^{n+1}}{1-z}$$

Learning Objectives

- Graph functions using vertical and horizontal shifts.
- Graph functions using reflections about the x-axis and the y-axis.
- Graph functions using vertical and horizontal scalings.
- Graph functions using a combination of transformations.

Transformations of Functions

A, B, C, D are all

Types of transformations from $y = f(x)$ to $y = Af(Bx - C) + D$

constants

Shifts

Vertical

$$h(x) = f(x) + D$$

①

Examples

$$y = x^2 + 2$$

if $D > 0$, shift up

if $D < 0$, shift down

Horizontal

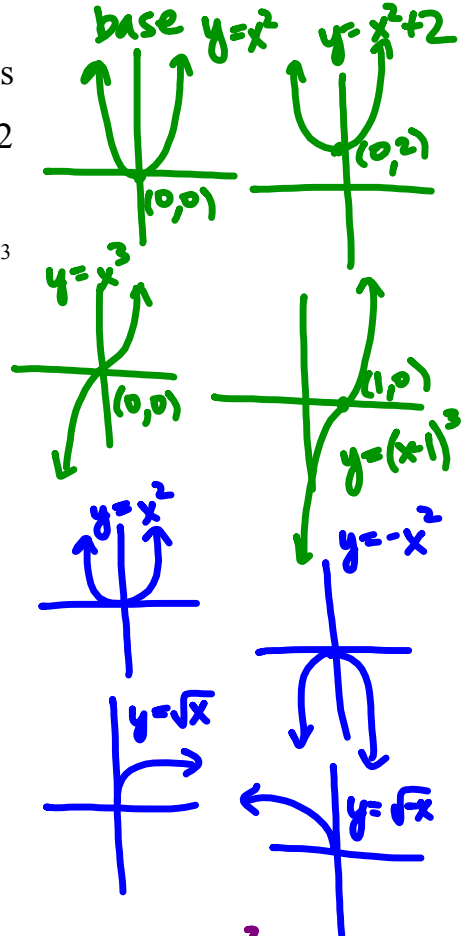
$$g(x) = f(x - C)$$

②

$$y = (x - 1)^3$$

$C = 1$

if $C > 0$, shift right
if $C < 0$, shift left



Reflect

Vertical

$$h(x) = -f(x)$$

③

$$y = -x^2$$

(across x-axis)

↑ outside the fn

Horizontal

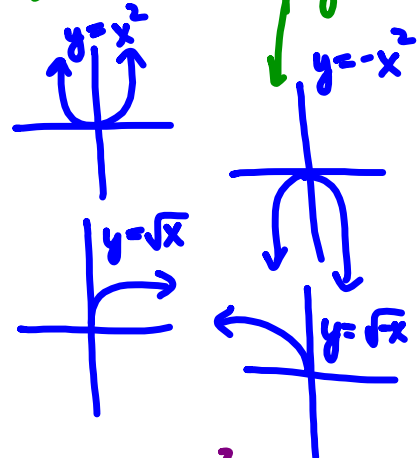
$$g(x) = f(-x)$$

④

$$y = \sqrt{-x}$$

(across y-axis)

↑ inside the fn



Stretch/shrink

(assume $A > 0, B > 0$)

Vertical

$$h(x) = A(f(x))$$

⑤

$$y = 5x^3$$

if $A < 1$, shrink
if $A > 1$, stretch

↑ outside the fn

Horizontal

$$g(x) = f(Bx)$$

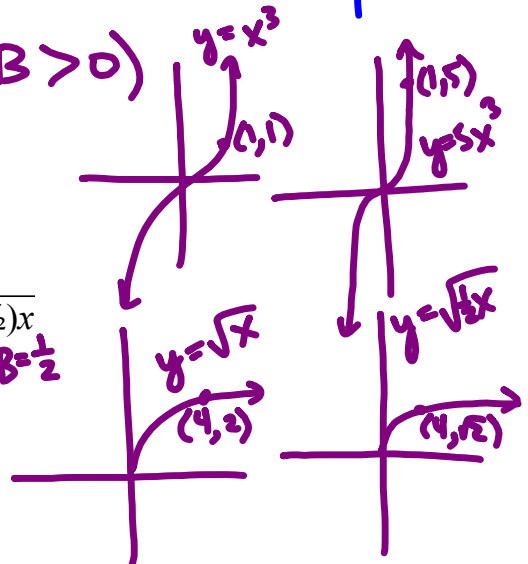
⑥

$$y = \sqrt{(1/2)x}$$

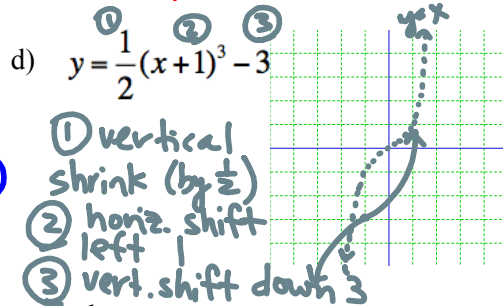
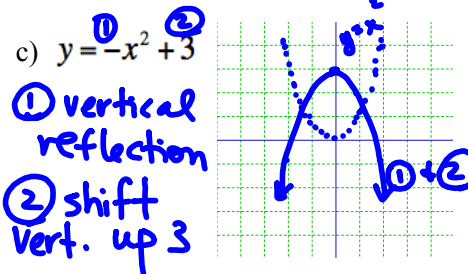
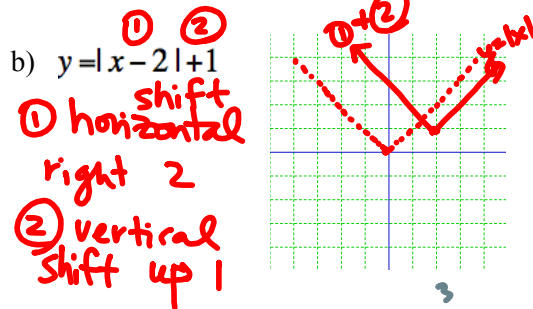
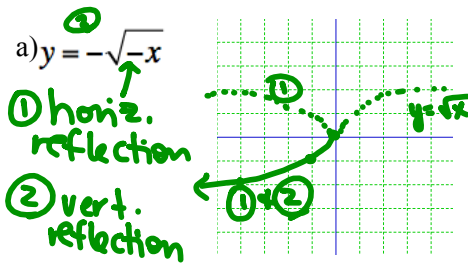
$B = 1/2$

if $B < 1$, stretch
if $B > 1$, shrink

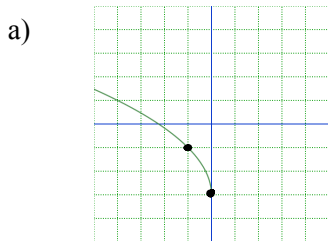
↑ inside the fn



Ex 1: Graph these functions.



Ex 2: Write an equation for each of these graphs.



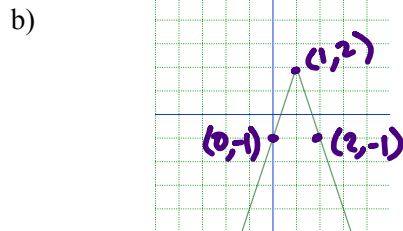
base fn: $y = \sqrt{x}$
our vertex at $(0, -3)$ (instead $(0, 0)$)
thru pt $(-1, -1)$ $(1, 1)$

① it's been reflected horizontally
 $\Rightarrow y = \sqrt{-x}$

② it's been stretched vertically by 2
 $\Rightarrow y = 2\sqrt{-x}$

③ it's been shifted down 3
 $\Rightarrow \boxed{y = 2\sqrt{-x} - 3}$

check $y(0) = 2\sqrt{0} - 3 = -3$
 $y(-1) = 2\sqrt{-1} - 3 = 2 - 3 = -1$



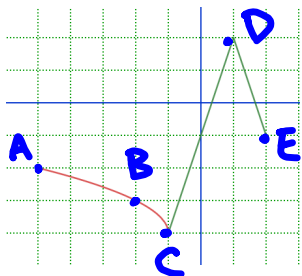
base fn: $y = |x|$

① vertical reflected
 $\Rightarrow y = -|x|$

② vertically stretched by 3
 $\Rightarrow y = -3|x|$

③ shifts
 $(0, 0) \rightarrow (1, 2)$
(a) horizontal shift right 1
(b) vertical shift up 2
 $\Rightarrow \boxed{y = -3|x-1| + 2}$

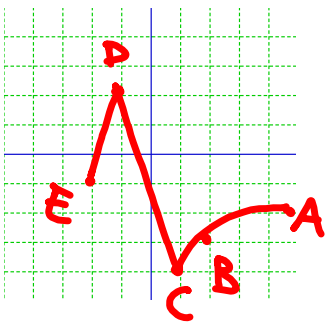
Ex 3: Given this graph for $f(x)$, sketch the graphs of the transformed functions.



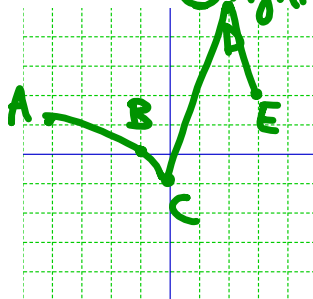
A (-5, 2)
 B (-2, -3)
 C (-1, -4)
 D (1, 2)

E (2, -1)

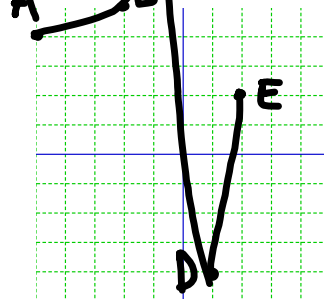
a) $f(-x)$ **horiz. ref.**



b) $f(x-1)+3$ **① up 3**
② right 1



c) $-2f(x)$ **① vert. ref.**
② vert. stretch by 2



Ex 4: Describe transformations compared to the base toolkit graph for each of these.

a) $f(x) = 2(x+1)^3 - 9$

base: $y = x^3$

- ① vertical stretch (by 2)
- ② horizontal shift left 1
- ③ vertical shift down 9

b) $f(x) = -2\sqrt{x+1} + 3$

base: $y = \sqrt{x}$

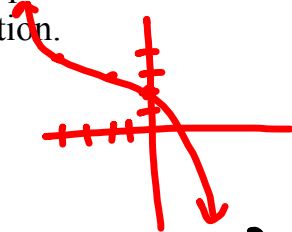
- ① vertical reflection
- ② vert. stretch (by 2)
- ③ horiz. shift left 1
- ④ vert. shift up 3

It may be helpful to use the table method to sketch a graph with several transformations. Let's look at a way to sketch this function.

$f(x) = \left(-\frac{1}{2}x - 1\right)^3 + 3$

base: $y = x^3$

$f(x) = \left(-\frac{1}{2}(x+2)\right)^3 + 3$



$y = x^3$	$y = -x^3$ ①	$y = \left(-\frac{1}{2}x\right)^3$ ②	$y = \left(-\frac{1}{2}(x+2)\right)^3$ ③	$y = \left(-\frac{1}{2}(x+2)\right)^3 + 3$ ④
(0, 0)	(0, 0)	(0, 0)	(-2, 0)	(-2, 3)
(1, 1)	(-1, 1)	(-2, 1)	(-4, 1)	(-4, 4)
(-1, -1)	(1, -1)	(2, -1)	(0, -1)	(0, 2)

horiz. refl horiz. stretch (by 2) h. shift L2 v. shift up 3

Ex 5: Use the table method above to sketch this function.

$f(x) = -3(x-2)^2 + 4$

base: $y = x^2$

$y = x^2$	$y = -(x^2)$	$y = -3x^2$	$y = -3(x-2)^2 + 4$
(0, 0)	(0, 0)	(0, 0)	(2, 4)
(1, 1)	(1, -1)	(1, -3)	(3, 1)
(-1, 1)	(-1, -1)	(-1, -3)	(1, 1)

v. reflectn v. stretch (by 3) shifts (a) R 2 (b) up 4

