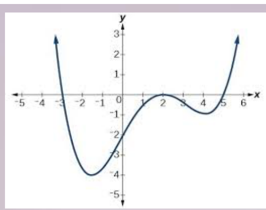

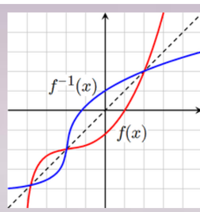

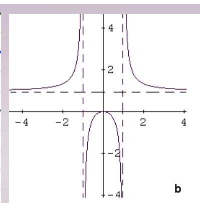

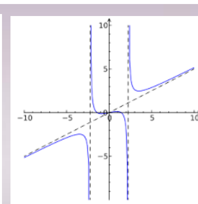

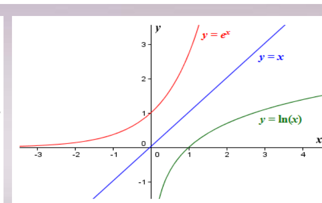

Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

4 Combinations of Functions

Learning Objectives

- Find and simplify functions involving arithmetic expressions.
- Combine functions through addition, subtraction, multiplication and
- division.
- Determine the domain of a function resulting from an arithmetic operation.
- Find the difference quotient of a function.
- Create a new function through composition of functions.
- Find the domain of a composite function.
- Find values of composite functions.
- Decompose a composite function into its component functions.

$(x-2)^2=(x-2)(x-2)=x^2-2x-2x+4=x^2-4x+4$

Ex 1: Evaluate this function at the given expressions, simplifying your answer.

f(x) = x^2 - 4*x* + 3 $\qquad f(\heartsuit)=\heartsuit^2-4\heartsuit+3$

a) *f*(-3)

$f(-3)=(-3)^2-4(-3)+3$

$=9+12+3$

$=\boxed{24}$

b) *f*(x-2) $\qquad \heartsuit=x-2$

$f(x-2)=(x-2)^2-4(x-2)+3$

$=x^2-4x+4-4x+8+3$

$=\boxed{x^2-8x+15}$

c) *f*(x^{-2}) $\qquad \heartsuit=x^{-2}$

$=(x^{-2})^2-4(x^{-2})+3$

$=\boxed{x^{-4}-4x^{-2}+3}$

d) *f*(x^2+1) $\qquad \heartsuit=x^2+1$

$=(x^2+1)^2-4(x^2+1)+3$

$=x^4+2x^2+1-4x^2-4+3$

$=\boxed{x^4-2x^2}$

It is also possible to perform arithmetic operations on functions.

Sum $\qquad f(x)+g(x)=(f+g)(x)$ (read this as "f plus g of x")

Difference $\qquad (f-g)(x)=f(x)-g(x)$

Product $\qquad (fg)(x)=f(x)\,g(x)$

Quotient $\qquad \left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$

Composition $\qquad (f\circ g)(x)=f(g(x))$ (read "f composed w/ g of x is f of g of x")

big open circle, NOT a multiplication sign

Ex 2: For $f(x)=\sqrt{x-1}$, and $g(x)=\frac{x}{x^2-4}$, simplify the resulting function and determine the domain if appropriate.

a) $(f+g)(x)$

$=f(x)+g(x)$

$=\sqrt{x-1}+\frac{x}{x^2-4}$

domain: $[1,2)\cup(2,\infty)$

① need $x-1\geq 0$

$\Leftrightarrow x\geq 1$

and ② need $x^2-4\neq 0$

$x^2\neq 4$

$x\neq \pm 2$

b) $(f-g)(5)$

$=f(5)-g(5)$

$=\sqrt{5-1}-\frac{5}{5^2-4}$

$=2-\frac{5}{21}=\frac{42}{21}-\frac{5}{21}$

$=\frac{37}{21}$

c) $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$

$=\frac{\sqrt{x-1}}{\frac{x}{x^2-4}}$

$=\sqrt{x-1}\div\frac{x}{x^2-4}$

$=\frac{\sqrt{x-1}\,(x^2-4)}{x}$

domain: $[1,2)\cup(2,\infty)$

① $x\neq 0$

and ② $x\neq\pm 2$

and ③ $x\geq 1$

$\Big\}\ x\geq 1,\ x\neq 2$

Ex 3: For the two functions $f(x)=\sqrt{x-1}$ and $g(x)=\frac{x}{x^2-4}$, find the following.

a) $f(g(x))$ $=f(g(x))$

work from "inside out"

$=f\left(\frac{x}{x^2-4}\right)$

$=\sqrt{\left(\frac{x}{x^2-4}\right)-1}$

b) $g(f(x))$

$=g(f(x))$

$=\frac{f(x)}{(f(x))^2-4}$

$=\frac{\sqrt{x-1}}{(\sqrt{x-1})^2-4}$

work from "outside in"

moral of story: in general

$f(g(x))\neq g(f(x))$.

Ex 4: For $f(x) = 3x + 5$, find $(f \circ f)(x)$ and its domain.

$$(f \circ f)(x) = f(f(x)) = f(3x+5)$$
$$= 3(3x+5) + 5 = 9x + 15 + 5 = 9x + 20$$

domain: $(-\infty, \infty)$

In calculus, one frequently is required to find a difference quotient, which is defined by $\boxed{\dfrac{f(x+h)-f(x)}{h}}$.

warning: $f(x+h) \neq f(x) + h$

order matters!

Ex 5: Find the difference quotient for each of these.

a) $f(x) = 3x + 5$

$$\frac{f(\boxed{x+h}) - f(x)}{h}$$
$$= \frac{3(\boxed{x+h}) + 5 - (3x+5)}{h}$$
$$= \frac{\cancel{3x} + 3h + \cancel{5} - \cancel{3x} - \cancel{5}}{h}$$
$$= \frac{3\cancel{h}}{\cancel{h}} = \boxed{3}$$

b) $f(x) = x^2 - 3x + 1$

$$\frac{f(\boxed{x+h}) - f(x)}{h}$$
$$= \frac{(\boxed{x+h})^2 - 3(\boxed{x+h}) + 1 - (x^2 - 3x + 1)}{h}$$
$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{3x} - 3h + \cancel{1} - \cancel{x^2} + \cancel{3x} - \cancel{1}}{h}$$
$$= \frac{2xh + h^2 - 3h}{h}$$
$$= \frac{\cancel{h}(2x + h - 3)}{\cancel{h}} = \boxed{2x + h - 3}$$

Decomposing Functions

Ex 6: Find two functions *f* and *g* such that $f(g(x)) = h(x)$ where

$h(x) = \frac{3}{(5x+1)^2}$.

1

$f(x) = \frac{3}{x}$ $g(x) = (5x+1)^2$

$\Rightarrow f(g(x)) = f\left((5x+1)^2\right) = \frac{3}{(5x+1)^2}$

2 $f(x) = \frac{3}{x^2}$ $g(x) = 5x+1$

check

$\Rightarrow f(g(x)) = \frac{3}{(g(x))^2} = \frac{3}{(5x+1)^2}$