



Math 1050 ~ College Algebra

8 Using Synthetic Division to Factor Polynomials

$$\begin{aligned} -3x + 4y &= 5 \\ 2x - y &= -10 \end{aligned}$$

$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^m k = \frac{m(m+1)}{2}$$

$$\sum_{k=0}^n z^k = \frac{1-z^{n+1}}{1-z}$$

Learning Objectives

- Use division to factor polynomials and determine zeros.
- Use synthetic division to simplify the division process.
- Use the Remainder Theorem to find function values of polynomials.
- Use the Factor Theorem to relate zeros to factors of polynomials.

When solving for the zeros of a function, it helps if we can break the function down into factors. Synthetic division will be useful to us.

Factor Theorem

A polynomial $f(x)$ has a factor $(x-k)$ if and only if $f(k) = 0$.

Remainder Theorem

If a polynomial $f(x)$ is divided by $(x-k)$, then the remainder $r = f(k)$.

Long division is ALWAYS useful for division of polynomials.

Synthetic division is only useful when dividing by $(x-k)$ where $k \in \mathfrak{R}$.

Ex 1: Use long division to divide $(4x^3 + 10x^2 - 2x - 5)$ by $(2x^2 - 1)$.

Ex 2: Divide $(x^3 + 4x^2 - 3x + 2)$ by $(x-3)$ in two ways.

Long division

Synthetic division

Ex 3: Use synthetic division to compute this quotient.

$$(5x^3 + 6x + 8) \div (x + 2)$$

Write the result in the form $f(x) = (x-k)(q(x)) + r(x)$

Ex 4: Use the remainder theorem and synthetic division to show that $(x+3)$ is a factor of this function.

$$f(x) = 3x^3 + 5x^2 - 3x + 27$$

Ex 5: Use division to show that $\frac{2}{3}$ is a solution of $48x^3 - 80x^2 + 41x - 6 = 0$.
Use the result to factor the polynomial completely and find all solutions.