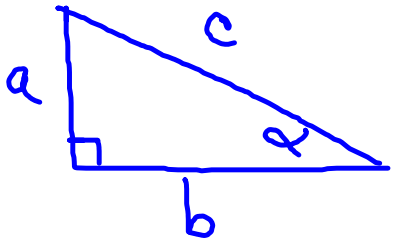
RIGHT TRIANGLE TRIGONOMETRY

In this lesson you will learn to:

- Evaluate trigonometric functions of acute angles in a right triangle.
- Use the fundamental trigonometric identities.
- Use a calculator to evaluate trigonometric functions.
- Use trigonometric functions to model and solve problems.



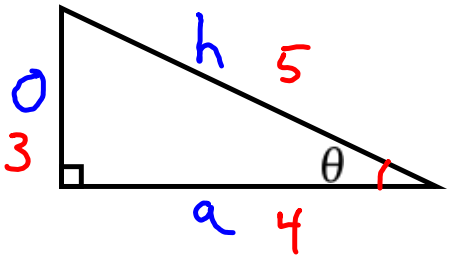
Right triangle with an acute angle θ.

Label the hypotenuse. h
Label the side opposite θ. o
Label the side adjacent to θ. a

$\sin\theta = \frac{o}{h}$ $\quad \csc\theta = \frac{h}{o}$

$\cos\theta = \frac{a}{h}$ $\quad \sec\theta = \frac{h}{a}$

$\tan\theta = \frac{o}{a}$ $\quad \cot\theta = \frac{a}{o}$



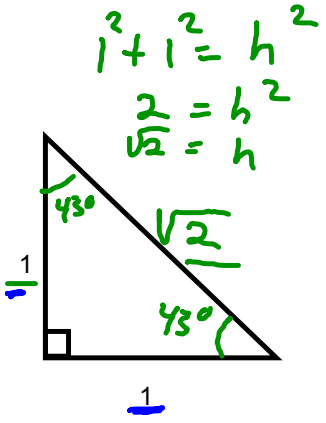
$3^2 + 4^2 = 5^2$

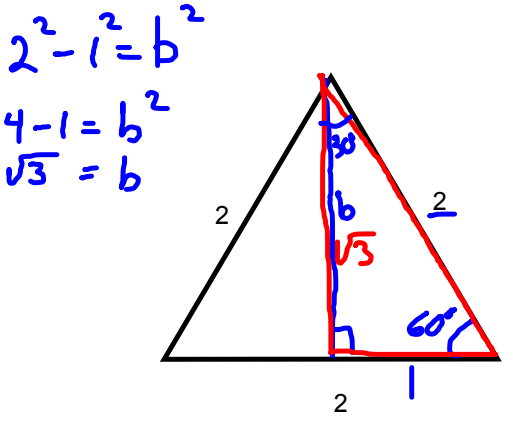
$9 + 16 = 25$

$\sin\theta = \frac{3}{5}$ $\quad \cos\theta = \frac{4}{5}$ $\quad \tan\theta = \frac{3}{4}$

$\csc\theta = \frac{5}{3}$ $\quad \sec\theta = \frac{5}{4}$ $\quad \cot\theta = \frac{4}{3}$

Let's verify with two special triangles:





sin 45° = $\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

cos 45° = $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

tan 45° = 1

sin 60° = $\frac{\sqrt{3}}{2}$ ↔ sin 30° = $\frac{1}{2}$

cos 60° = $\frac{1}{2}$ ↔ cos 30° = $\frac{\sqrt{3}}{2}$

tan 60 ° = $\frac{\sqrt{3}}{1}=\sqrt{3}$ ↔ tan 30 ° = $\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

Cosine means Sine of the Complement.

Notice that co-functions of complementary angles are equal.

$\sin(90^\circ - \theta) = \cos\theta$

$\tan(90^\circ - \theta) = \cot\theta$

$\sec(90^\circ - \theta) = \csc\theta$

Trigonometric identities:

Reciprocal Identities:

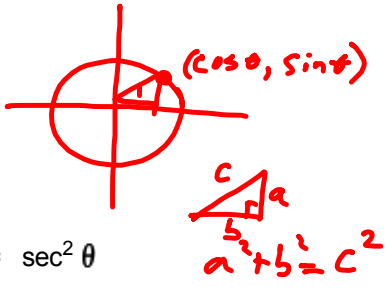
$\sin\theta = \frac{1}{\csc\theta}$ $\quad \csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\quad \sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\quad \cot\theta = \frac{1}{\tan\theta}$

Quotient Identities:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$



Pythagorean Identities:

✓ $\sin^2\theta + \cos^2\theta = 1$

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$1 + \cot^2\theta = \csc^2\theta$ → $1 + \cot^2\theta = \csc^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

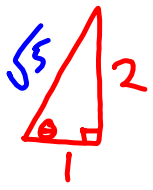
$\tan^2\theta + 1 = \sec^2\theta$ → $1 + \tan^2\theta = \sec^2\theta$

Example 1:

Two ways to get information:

Right triangle θ is acute

If $\tan\theta = 2$, find $\sec\theta$.



$h^2 = 2^2 + 1^2$

$h^2 = 4 + 1$

$= 5$

$h = \pm\sqrt{5}$

$\sec\theta = \frac{\sqrt{5}}{1} = \sqrt{5}$

Use identities

$1 + \tan^2\theta = \sec^2\theta$

$1 + 2^2 = \sec^2\theta$

$5 = \sec^2\theta$

$\pm\sqrt{5} = \sec\theta$

Using a calculator to evaluate trigonometric functions.

Use your calculator (be careful of the mode, radians or degrees) to determine these:

sin 35° $\approx .57$

cos 75° $\approx .26$

tan 20° $\approx .36$

sec 24° $= \frac{1}{\cos 24°}$ $(\cos 24°)^{-1} \approx 1.09$

cot 15° ≈ 3.73

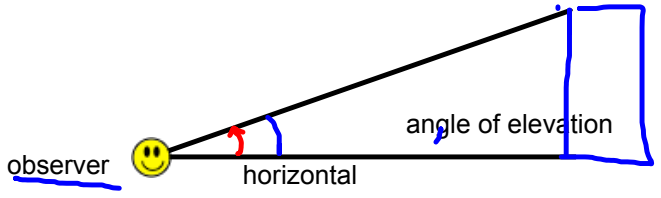
csc 80° ≈ 1.02

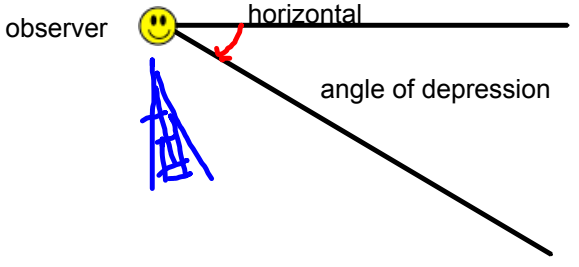
tan $\pi/8$ $\approx .414$

sec $3\pi/5$ ≈ -3.24

csc $2\pi/5$ ≈ 1.05

Angles when sighting





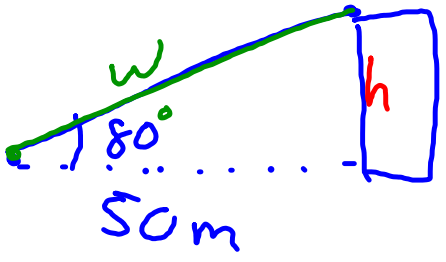
Example 2:

Solving problems using right triangles.

- Draw a picture.
- Label known parts.
- Use trigonometric functions to determine desired parts.

If you are standing 50 meters from the base of a tall building and you determine the angle of elevation to the top of the building to be 80º.

a. Determine the height of the building.



$\tan 80° = \frac{h}{50\text{ m}}$

$h = 50 \cdot \tan 80°$

$h \approx 283.56\text{ m}$

b. If you wish to string a wire from you to the top of the building, how long does the wire need to be?

$\cos 80° = \frac{50}{w}$

$w = \frac{50}{\cos 80°} \approx 287.94\text{ m}$