

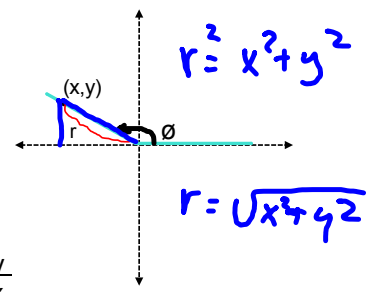
## 1.4 ~ Trig Functions of Any Angle

- \* Use reference angles to evaluate trigonometric functions of any angle.
- \* Evaluate trigonometric functions of real numbers.

The trigonometric functions (sine, cosine, tangent, etc.) can be associated with any angle, even non-acute angles.

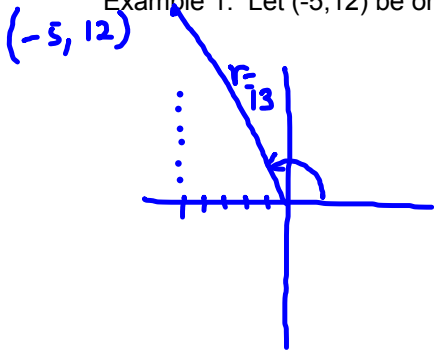
For any angle in standard position with  $(x,y)$  on the terminal side and  $r = \sqrt{x^2 + y^2} \neq 0$ ,

$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}$
$\csc \theta = \frac{r}{y}$	$\sec \theta = \frac{r}{x}$	$\cot \theta = \frac{x}{y}$



This allows us to find the functions of any angle.

Example 1: Let  $(-5, 12)$  be on the terminal side of  $\theta$ . Find the sine, cosine and tangent of  $\theta$ .



$$\begin{aligned}r^2 &= (-5)^2 + 12^2 \\ &= 25 + 144 \\ &= 169\end{aligned}$$

$$r = 13$$

$$\sin \theta = \frac{12}{13}$$

$$\cos \theta = -\frac{5}{13}$$

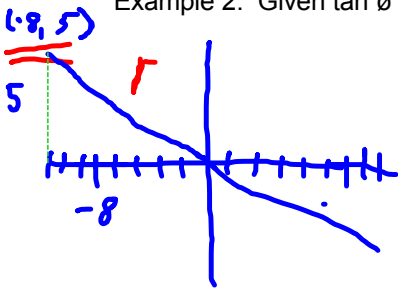
$$\tan \theta = -\frac{12}{5}$$

$$\csc \theta = \frac{13}{12}$$

$$\sec \theta = -\frac{13}{5}$$

$$\cot \theta = -\frac{5}{12}$$

Example 2: Given  $\tan \theta = -5/8$  and  $\cos \theta < 0$ , find  $\sec \theta$  and  $\sin \theta$ .



(-8, 5)

$$\frac{y}{x} = \frac{5}{-8}$$

$$r^2 = (-5)^2 + (8)^2$$
$$= 25 + 64$$

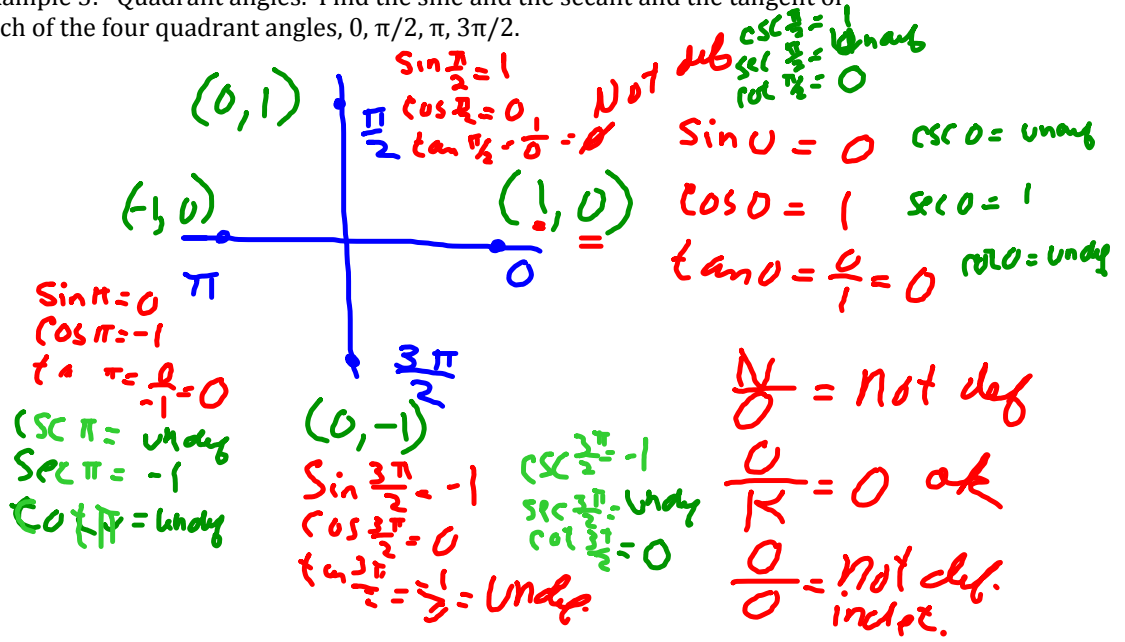
$$\cos \theta = \frac{-8}{r} = 89$$

$$\cos \theta = \frac{-8}{\sqrt{89}} \quad r = \sqrt{89}$$

$$\sec \theta = \frac{-\sqrt{89}}{8}$$

$$\sin \theta = \frac{5}{\sqrt{89}}$$

Example 3: Quadrant angles. Find the sine and the secant and the tangent of each of the four quadrant angles,  $0, \pi/2, \pi, 3\pi/2$ .



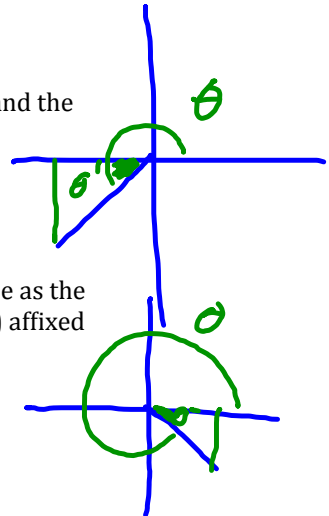
## REFERENCE ANGLES

A reference angle is the angle made by the terminal side of an angle and the horizontal axis. The reference angle of  $\theta$  is called  $\theta'$ .

It is positive.  
It is acute.

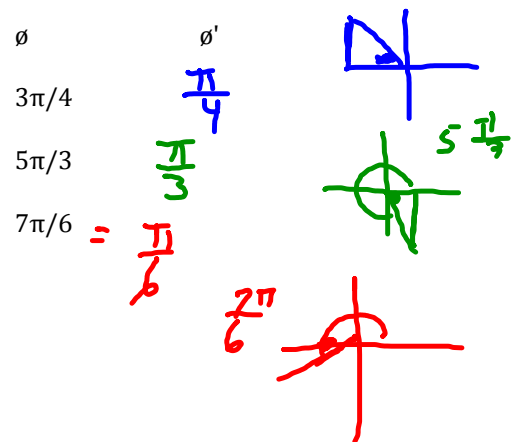
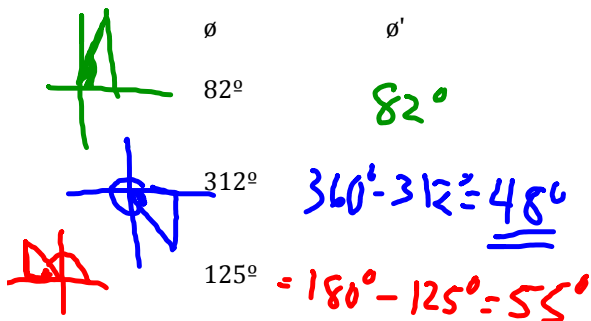
The trigonometric functions of any angle will be numerically the same as the respective functions of the reference angle with the proper sign (+/-) affixed according to the quadrant the terminal side lies in.

If the angle is in degrees, the reference angle will be in degrees.  
If the angle is in radians, the reference angle will be in radians.

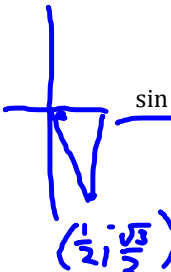


Find the reference angle for each of these:

*any acute angle is its own ref. angle*



Now we can find the functions of any angle or for that matter, any real number.

$$\sin 35\pi/3 = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$



$(\frac{1}{2}, \frac{\sqrt{3}}{2})$

$$2\pi = \frac{6\pi}{3}$$

$$\frac{35\pi}{3} - \frac{30\pi}{3} = \frac{5\pi}{3}$$

$$\cos 25\pi/7 = \cos 3\pi/7$$

$$-\frac{14\pi}{7} \text{ cotermin.}$$

$$2\pi - 4\pi = 3\pi/7 \text{ ref angle}$$


$$\tan 495^\circ = \tan 135^\circ = -\tan 45^\circ = -1$$

$$-360$$

$$135^\circ \text{ cotermin}$$

$$45^\circ \text{ ref angle.}$$
