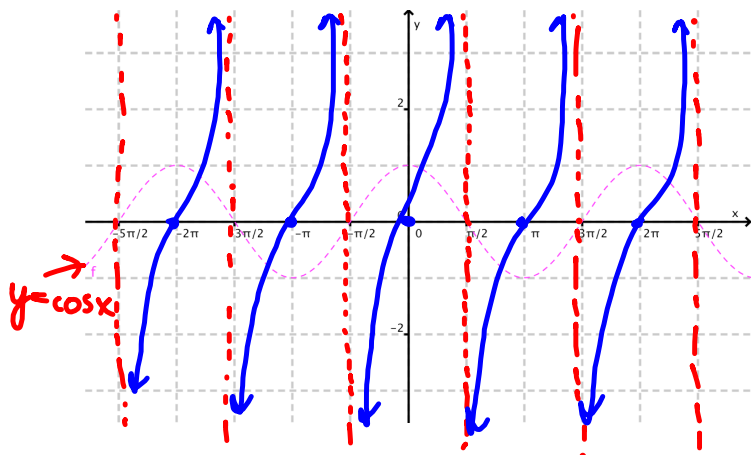


1.6 ~ Graphs of Other Trigonometric Functions

You will learn to:

- Sketch graphs of tangent and cotangent functions.
- Sketch graphs of secant and cosecant functions.
- Analyze the transformations of these functions.

$$f(x) = \tan x = \frac{\sin x}{\cos x}$$



\mathbb{R} = real #s
 \mathbb{Z} = integers
 Asymptotes: $\cos x = 0$

\in element of
 of

Period: π

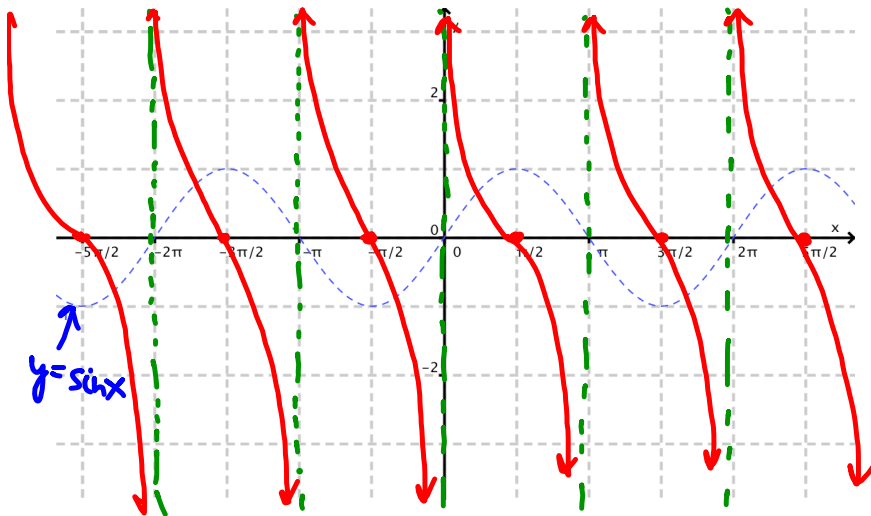
Domain: $x \in \mathbb{R}$,
 $x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$

Range: $y \in \mathbb{R}$

Asymptotes:

$$x = \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$$

$$f(x) = \cot x = \frac{\cos x}{\sin x}$$



Period: π

Domain: $x \in \mathbb{R},$
 $x \neq n\pi, n \in \mathbb{Z}$

Range: $(-\infty, \infty)$

vertical

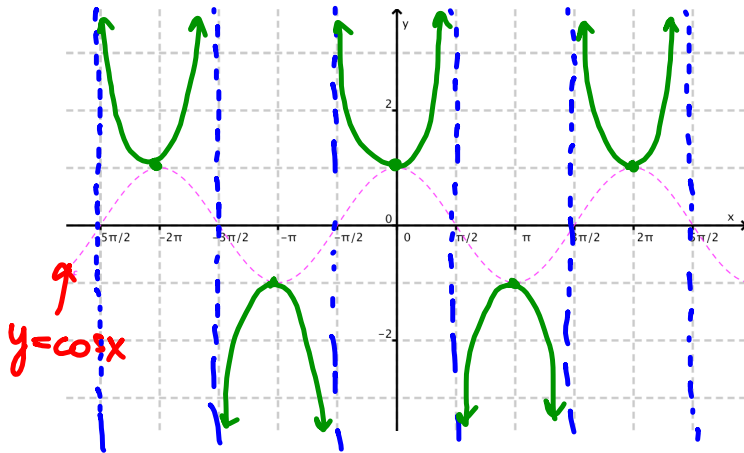
Asymptotes:

$x = n\pi, n \in \mathbb{Z}$

$$f(x) = \sec x = \frac{1}{\cos x}$$

Sketch $y = \cos x$

Then plot asymptotes and points on $y = \sec x$



Period: 2π

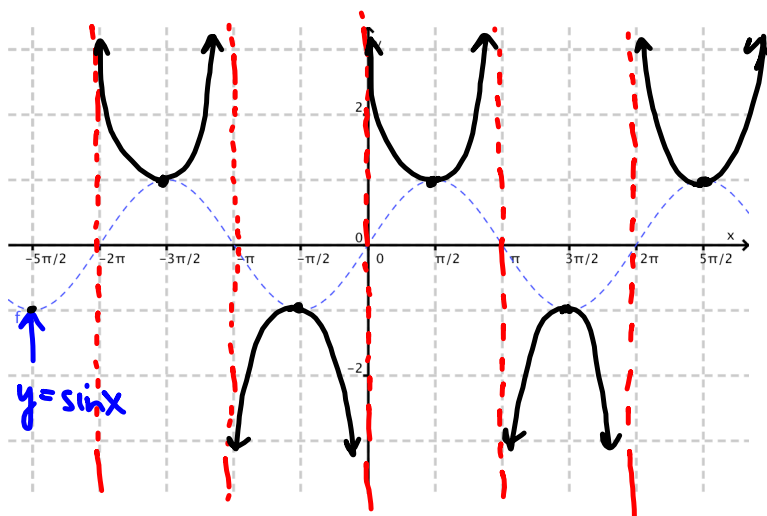
Domain: $x \in \mathbb{R}$
 $x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$

Range: $y \in (-\infty, -1] \cup [1, \infty)$

vertical
Asymptotes:

$x = \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$

$$f(x) = \csc x = \frac{1}{\sin x}$$



Period: 2π

Domain: $x \in \mathbb{R}, x \neq n\pi, n \in \mathbb{Z}$

Range: $y \in (-\infty; -1] \cup [1; \infty)$

Asymptotes:

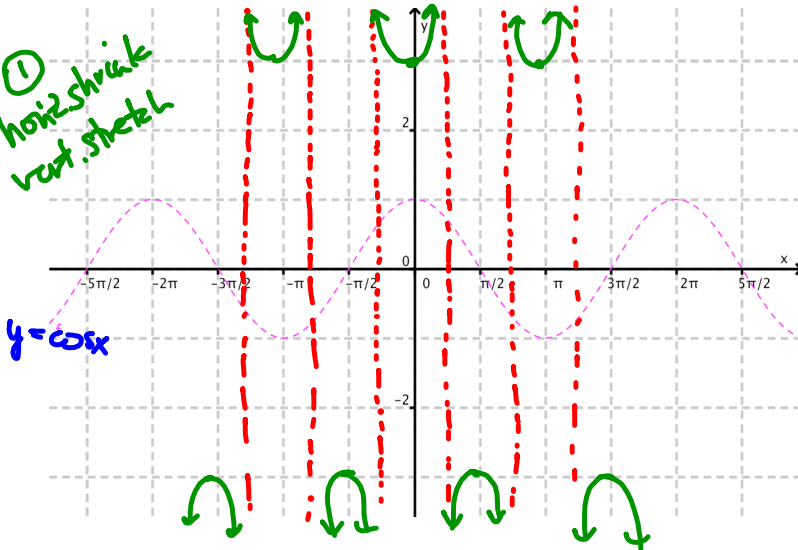
$x = n\pi, n \in \mathbb{Z}$

Example 1: Graph this function with transformations.

$$f(x) = 3 \sec(2x) + 1$$

vertical stretch
horizontal shrink (by $\frac{1}{2}$)
shift up 1

①
- horiz shrink
- vert stretch



$y = \sec x$ has
period of 2π

Period: $\frac{2\pi}{2} = \pi$

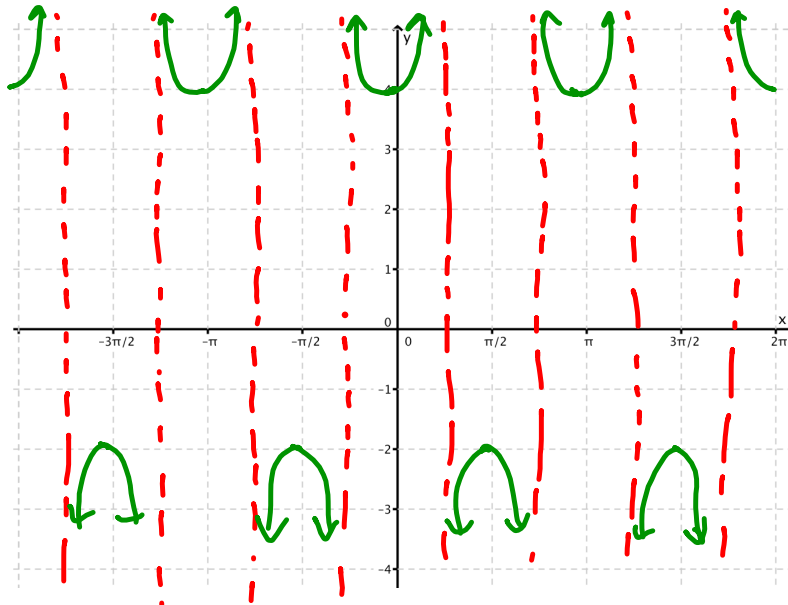
Domain: $x \in \mathbb{R}, x \neq \frac{(2n+1)\pi}{4}$

Range: $(-\infty, -3] \cup [3, \infty)$

vertical
Asymptotes:

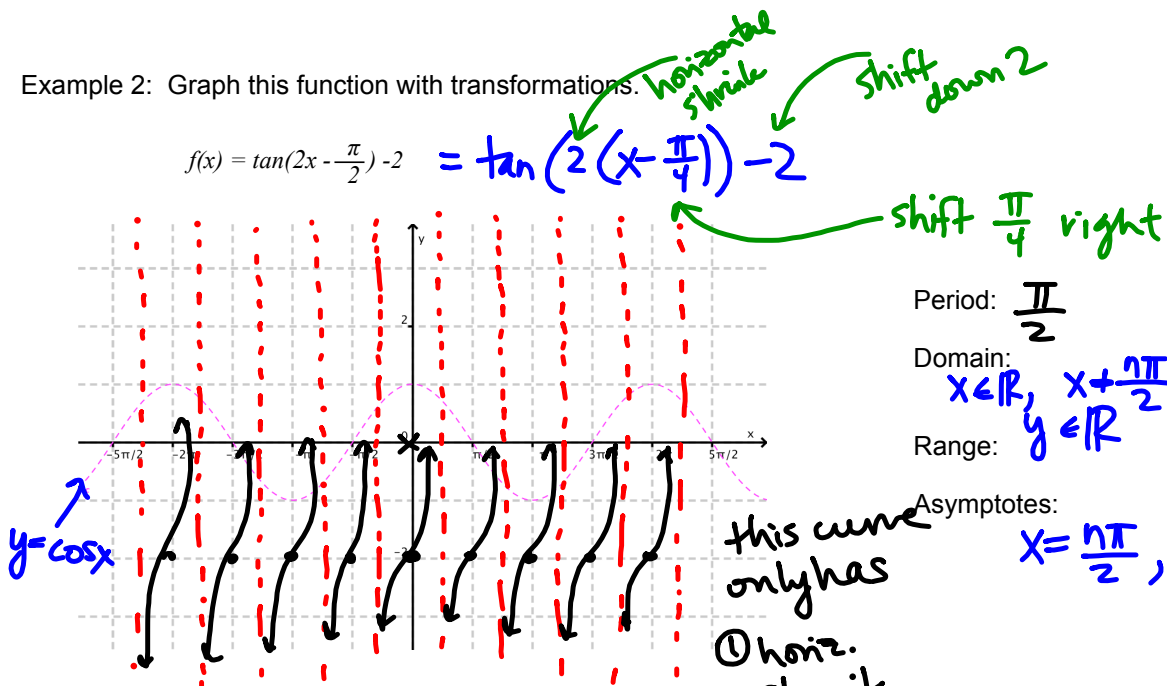
$$x = \pm \frac{\pi}{4}, \pm \frac{3\pi}{4}, \pm \frac{5\pi}{4}, \dots$$

$$\text{or } x = \frac{(2n+1)\pi}{4}, n \in \mathbb{Z}$$



Example 2: Graph this function with transformations.

$$f(x) = \tan\left(2x - \frac{\pi}{2}\right) - 2 = \tan\left(2\left(x - \frac{\pi}{4}\right)\right) - 2$$



Period: $\frac{\pi}{2}$

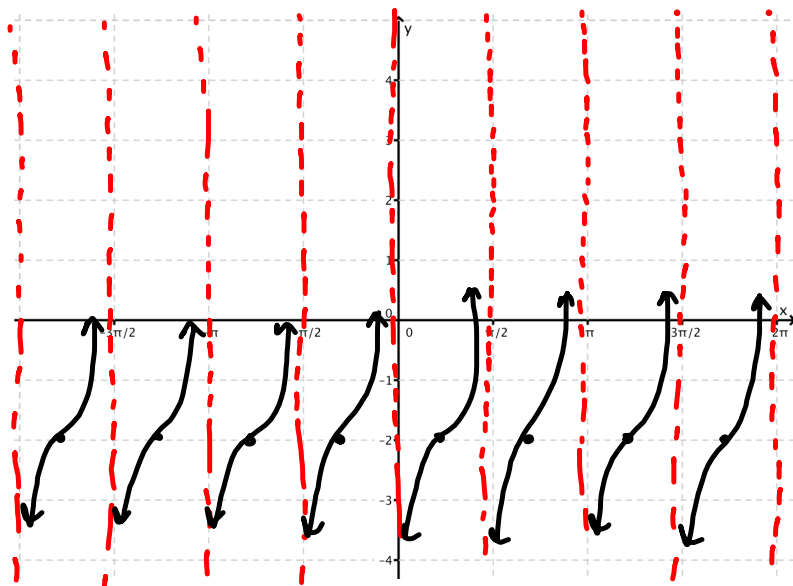
Domain: $x \in \mathbb{R}, x \neq \frac{n\pi}{2}$

Range: $y \in \mathbb{R}$

Asymptotes:

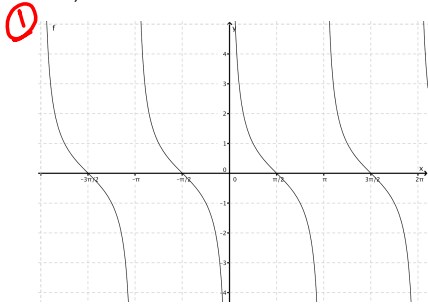
$$x = \frac{n\pi}{2}, n \in \mathbb{Z}$$

this curve only has
 ① horiz. shrink
 and ② vert. shift



Example 3:

- a) Write an equation for each of these graphs, assuming there are no transformations.
 b) Write each function that is a co-function as a transformation of another function.



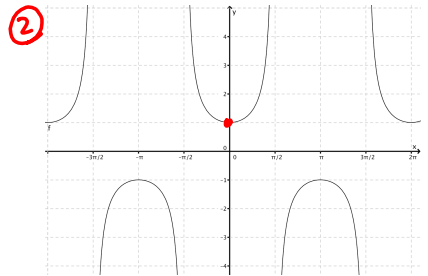
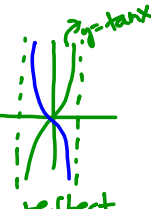
(a) $y = \cot x$

(b) $y = \tan\left(-\left(x - \frac{\pi}{2}\right)\right)$

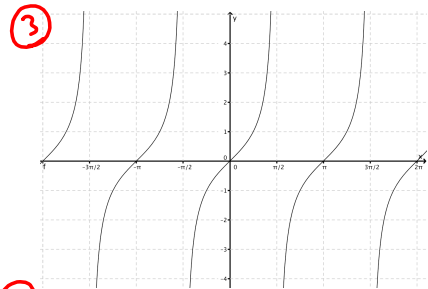
OR $y = -\tan\left(x - \frac{\pi}{2}\right)$

($\tan x = f(x)$
is odd fn)

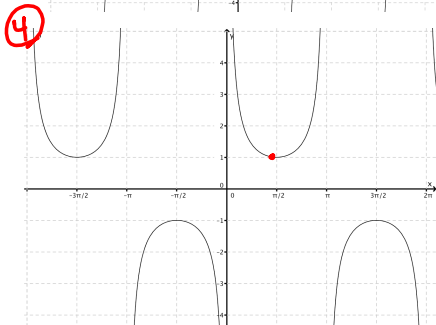
• horiz. reflect
then shift to
right by $\frac{\pi}{2}$



(a) $y = \sec x$



(a) $y = \tan x$



(a) $y = \csc x$

(b) same shape as $y = \sec x$
but shifted

right by $\frac{\pi}{2}$

$y = \sec\left(x - \frac{\pi}{2}\right)$

$\Rightarrow \sec\left(x - \frac{\pi}{2}\right) = \csc x$

Note

odd fns:	$y = \sin x$	$y = \csc x$	$f(-x) = -f(x)$
	$y = \tan x$	$y = \cot x$	
even fns:	$y = \cos x$	$y = \sec x$	$f(-x) = f(x)$

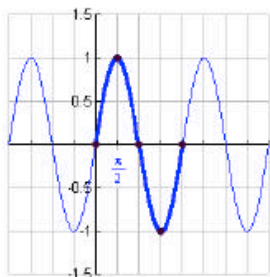
Graphs of the Six Trigonometric Functions

$y = \sin x$

Domain:
All Reals

Range:
[-1, 1]

Period: 2π



$y = \cos x$

Domain:
All Reals

Range:
[-1, 1]

Period: 2π

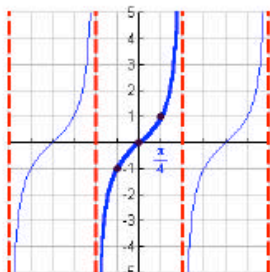


$y = \tan x$

Domain:
All $x \neq \frac{\pi}{2} + n\pi$

Range:
All Reals

Period: π

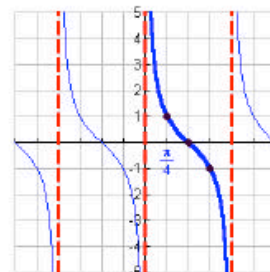


$y = \cot x$

Domain:
All $x \neq n\pi$

Range:
All Reals

Period: π

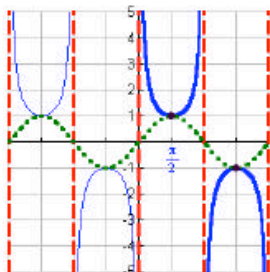


$y = \csc x$

Domain:
All $x \neq n\pi$

Range:
 $(-\infty, -1] \cup [1, \infty)$

Period: 2π



$y = \sec x$

Domain:
All $x \neq \frac{\pi}{2} + n\pi$

Range:
 $(-\infty, -1] \cup [1, \infty)$

Period: 2π

