2.2 ~ Verifying Trigonometric Identities

Rules:

* Work on one side at a time.
* Factor, add, multiply as needed.
* If all else fails, change everything to sines and cosines.
* Never multiply or divide both sides by something.
* Always try something.

Example 1: Verify this identity
Prove

$$\cos x + \sin x \tan x = \sec x \quad \text{goal}$$

Pf $\quad \cos x + \sin x \tan x$

$$= \cos x + \sin x \left(\frac{\sin x}{\cos x} \right)$$

$$= \cos x \left(\frac{\cos x}{\cos x} \right) + \frac{\sin^2 x}{\cos x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x \quad //$$

Example 2: Verify

$\sec x - \cos x = \sin x \tan x$ goal

Pf $\sec x - \cos x$

$$= \frac{1}{\cos x} - \cos x \left(\frac{\cos x}{\cos x} \right)$$

$$= \frac{1 - \cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$= \sin x \left(\frac{\sin x}{\cos x} \right)$$

$$= \sin x \tan x$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

Example 2: Verify

sec x - cos x = sin x tan x

$$\underline{Pf} \quad \sec x - \cos x$$

$$= \frac{1}{\cos x} - \cos x \left(\frac{\cos x}{\cos x} \right)$$

$$= \frac{1 - \cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

Rationalizing the denominator by using a conjugate is powerful.

Example 3: Verify

$$\frac{\sec x - 1}{1-\cos x} = \boxed{\sec x} \quad \text{goal}$$

conjugate

$$(a-b) \rightarrow (a+b)$$

instead

$$\left(\frac{\sec x - 1}{1-\cos x}\right)\left(\frac{\cos x}{\cos x}\right)$$

$\underline{Pf}$

$$\left(\frac{\sec x - 1}{1-\cos x}\right)\left(\frac{1+\cos x}{1+\cos x}\right)$$

$$= \frac{\sec x + \sec x \cos x - 1 - \cos x}{1 - \cos^2 x}$$

$$\boxed{\begin{array}{c} \sec x \cos x \\ = \left(\frac{1}{\cos x}\right)\cos x = 1 \end{array}}$$

$$= \frac{\sec x - \cos x}{1-\cos^2 x}$$

$$= \left(\frac{\frac{1}{\cos x} - \cos x}{1 - \cos^2 x}\right)\left(\frac{\cos x}{\cos x}\right)$$

$$= \frac{(1 - \cos^2 x)}{(1-\cos^2 x)\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x \quad //$$

Example 4: Verify

$$\frac{\cot^2 x}{1 + \csc x} = \frac{1 - \sin x}{\sin x}$$

$\underline{Pf} \quad \dfrac{\cot^2 x}{1 + \csc x}$

$$= \left(\frac{\dfrac{\cos^2 x}{\sin^2 x}}{1 + \dfrac{1}{\sin x}} \right) \left(\frac{\sin^2 x}{\sin^2 x} \right)$$

$$= \frac{\cos^2 x}{\sin^2 x + \sin x}$$

$$= \frac{\cos^2 x}{\sin x (\sin x + 1)}$$

$$= \frac{1 - \sin^2 x}{\sin x (\sin x + 1)}$$

$$= \frac{(1 + \sin x)(1 - \sin x)}{\sin x (1 + \sin x)}$$

$$= \frac{1 - \sin x}{\sin x}$$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$