

## 2.4 ~ Sum and difference formulas

- Develop and use sum and difference formulas.
- Evaluate trigonometric functions using these formulas.
- Verify identities using these formulas.
- Solve more trigonometric equations.

Which are true and which are false?

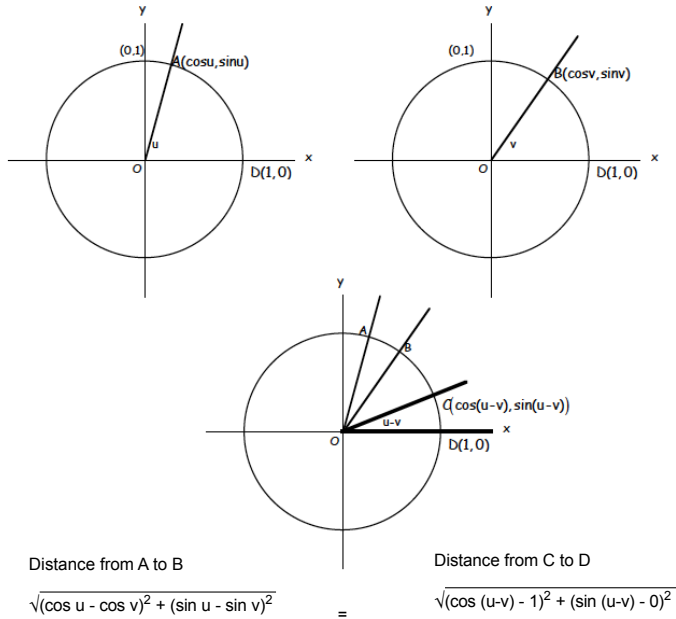
$$5(c+d) = 5c + 5d$$

$$\frac{a+b}{2} = \frac{a}{2} + \frac{b}{2}$$

$$(x+y)^2 = x^2 + y^2$$

$$\sqrt{p+q} = \sqrt{p} + \sqrt{q}$$

$$\sin(u+v) = \sin u + \sin v$$



Simplify each side of this equality to arrive at the formula on the next page.

Simplifying the equation on the previous page gives us one of our sum/difference formulas for trigonometric expressions.

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

In a similar way, we can derive these formulas:

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

These are the formulas for the sum/difference of a tangent.

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Why do we need to know these?

Example 1:

a) Find the exact value of  $\cos 75^\circ$ .

$$\cos 75^\circ = \cos (45^\circ + 30^\circ)$$

According to the formula this is:

$$\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

b)  $\sin \left( \frac{5\pi}{12} \right)$

Example 2: Verify this identity.

$$\sin \left( \frac{\pi}{2} - x \right) = \cos x$$

Example 3:

Solve this equation for all  $x$  on the interval  $[0, 2\pi)$ .

$$\sin\left(x + \frac{\pi}{4}\right) + \cos\left(x + \frac{\pi}{4}\right) = 1$$

Example 4: Now one harder one:

Simplify (state as an algebraic expression in terms of  $x$ ).

$$\sin(\arctan(2x) + \arccos x)$$