Extra ~ Review of Inverse Functions

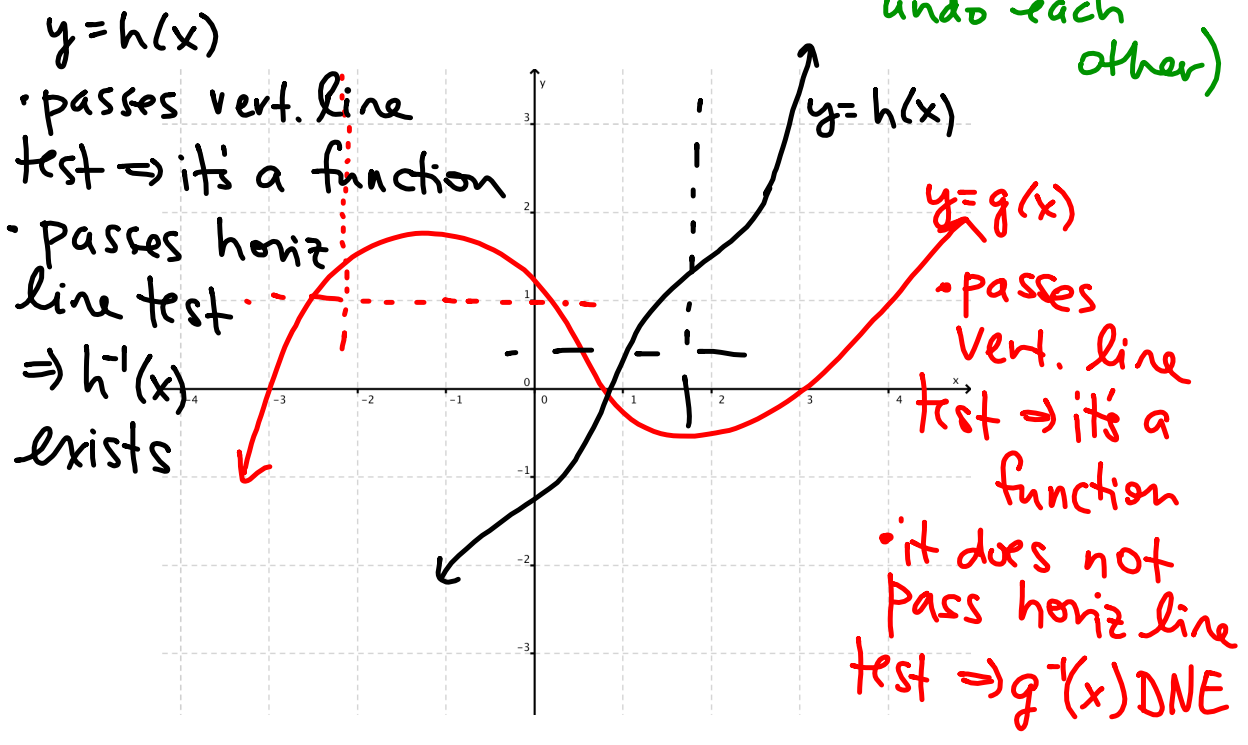
You will learn to:

- Determine whether a function has an inverse.
- Find and verify the inverse function if there is one.
- Sketch a function and its inverse.

Reminders About a Function and Its Inverse

The inverse of a function, $f(x)$, is written $f^{-1}(x)$ (read *f*-inverse).

The *-1* is NOT an exponent.
The original function must be *1-to-1*. (passes horizontal & vertical line tests)
The graph $y = f^{-1}(x)$ (the inverse function) is a reflection of $y = f(x)$ across the line $y = x$.
An (a,b) pair on the function becomes a (b,a) pair on the inverse.
$f(f^{-1}(x)) = x$ for every x in the domain of $f^{-1}(x)$, and vice versa. (f and f-1 "undo" each other)
The domain of $f^{-1}(x)$ is the range of $f(x)$ and vice versa.



Some questions about a familiar function:

What is the square root of 4? $\sqrt{4} = 2$

What number(s) can I square to get 4? ± 2

- $x^2 = 4$, so $x = ?$ ± 2
- $\sqrt{4} = ?$ 2

What is the *principal* square root of 4? 2

If $x = -3$, then $\sqrt{x^2} =$ $\sqrt{(-3)^2} = \sqrt{9} = 3 \neq -3$

If $x = -3$, then $(\sqrt{x})^2 =$ $(\sqrt{-3})^2$ not possible

so, $\sqrt{x^2} = |x|$

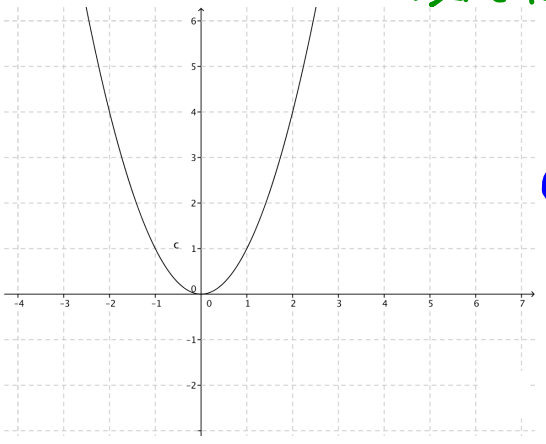
and $(\sqrt{x})^2 =$ x (assuming that $x \geq 0$)

Squaring undoes sq. root but sq. root does not undo squaring.

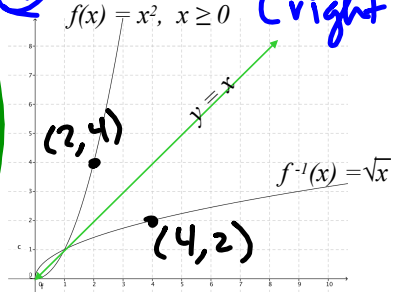
moral of the story:

$\sqrt{x^2} = |x| \neq x$

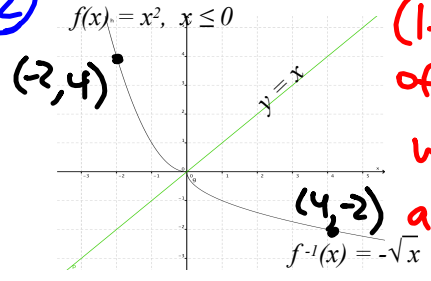
i.e. square root fn and square fn do NOT always undo each other!



(it passes vert. line test but not horiz. line test)

① 

(right half of parabola) which does pass horiz. line test

② 

(left half of parabola) which does also pass horiz. line test

Mostly, we choose scenario ① to be the inverse curve.

check (for ①) $f(x) = x^2,\ x \geq 0$, $f^{-1}(x) = \sqrt{x}$

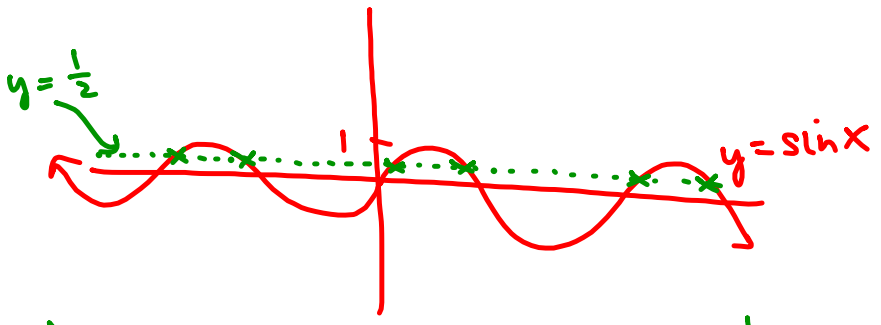
$f(f^{-1}(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$

(for ②) $f(x) = x^2,\ x \leq 0$, $f^{-1}(x) = -\sqrt{x}$

$f(f^{-1}(x)) = f(-\sqrt{x}) = (-\sqrt{x})^2 = x$

As we determine inverses of our trigonometric functions, this is why

$\sin x = 0.5$ has many solutions for x, and $\sin^{-1}(0.5) = ?$ has only one answer.



$\Rightarrow$ $y = \sin x$ does not pass horiz line test

$\Rightarrow$ $\sin x = f(x)$ is NOT invertible unless we restrict domain of $f(x) = \sin x$.