

Math 1060 ~ Trigonometry

Review of the Concept of Inverse Functions

Learning Objectives

In this section you will:

- Determine whether a function has an inverse.
- Use correct notation to write the inverse of a function.
- Find and verify the inverse if there is one.
- Sketch a function and its inverse.

$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2 \sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

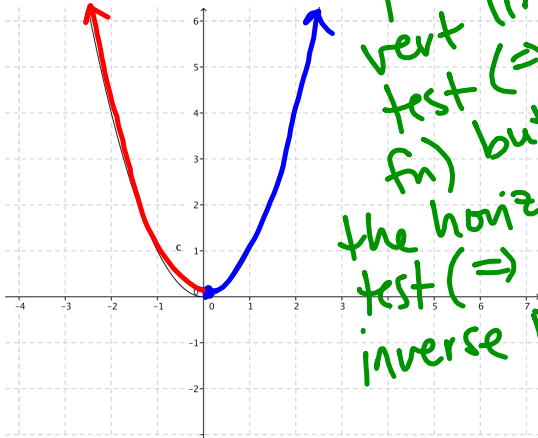
Reminders About a Function and Its Inverse

- The inverse of a function, $f(x)$, is written $f^{-1}(x)$ (read "f-inverse"). (not a reciprocal)
- The -1 is NOT an exponent.
- The original function must be 1-to-1. (passes vert. and horiz. line test)
- The graph of $y = f^{-1}(x)$ (the inverse function) is a reflection of $y = f(x)$ across the line $y = x$.
- An (a, b) point on the graph of the function becomes a (b, a) point on the graph of the inverse function.
- The domain of $f^{-1}(x)$ is the range of $f(x)$ and vice versa.
- $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ for every x in the domain of f .

(i.e. colloquially, we could say f and f^{-1} undo each other)

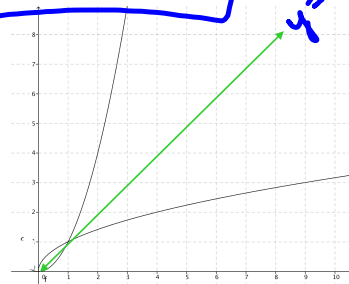
Let's demonstrate this with $f(x) = x^2$ and its inverse function.

$$f(x) = x^2$$

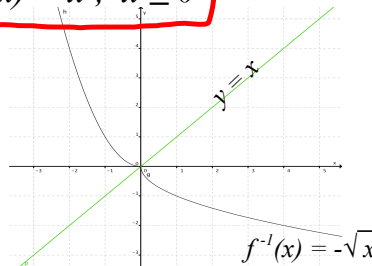


passes vert line test (\Rightarrow is a fn) but not the horiz. line test (\Rightarrow its inverse DNE)

$$f(x) = x^2, x \geq 0$$



$$f(x) = x^2, x \leq 0$$



For the quadratic function, $f(x) = x^2, x \geq 0$,
the inverse function is $f^{-1}(x) = \sqrt{x}$


Ex 1: Answer each of these:

- a) What number can I square to get 4? $2, -2$
- b) If $x^2 = 4$, then $x = 2$ or -2
- c) $\sqrt{4} = 2$
- d) What is the principal square root of 4? 2
- e) List all square roots of four. 2 or -2

Notice that the way the question is asked determines the number of answers.

Thus, when we develop inverses for the trigonometric functions, we must consider this.

$\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$



(or any other coterminal angles)

$\sin^{-1}\left(-\frac{1}{2}\right) = \arcsin\left(-\frac{1}{2}\right) = ? \quad -\frac{\pi}{6}$

(there can only be ONE answer)