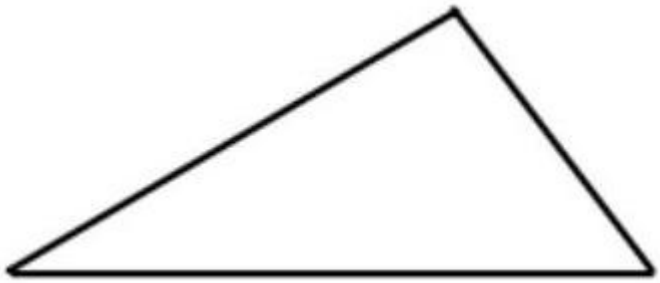
Math 1060 ~ Trigonometry

15 The Law of Sines

We will now apply our techniques to oblique triangles, those with no right angle.

It is important to label sides and angles of a triangle in a specific way.

Label the vertices A, B, C and the sides opposite them a, b, c respectively and the angles α, β, γ respectively.



The Law of Sines states that given any triangle ABC, $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$.
It may also be stated this way: $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$

We will prove it here.

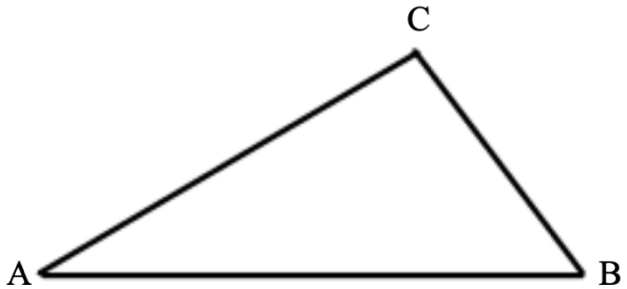
Given: $\triangle ABC$

Prove: $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$

Draw altitude $\overline{CD} \perp \overline{AB}$
Let $CD = h$

In $\triangle ACD, \sin \alpha =$

In $\triangle BCD, \sin \beta =$
Solve each for h and set them equal to each other.

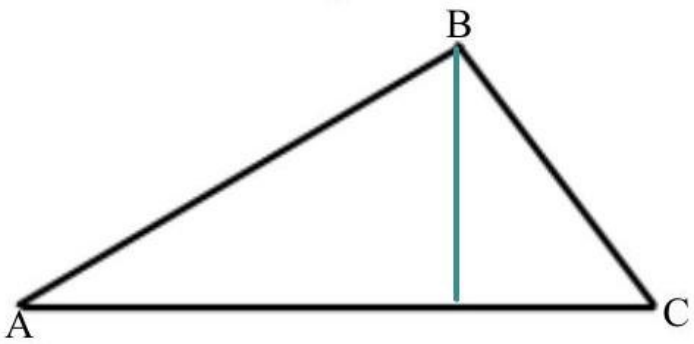


Area of a Triangle:

There are two alternate formulas for the area of a triangle.

We will prove the first one.

$A = \frac{1}{2}ab\sin\ \gamma$



EX 1

Given triangle KLM, with $m = 6$ cm and the angle at L measuring 40° and the angle at K measuring 75°, solve for the remaining parts of the triangle and find the area.

EX 2

Given triangle PQR , with the angle at P measuring 120°, the angle at Q measuring 30° and $p = 10$ft, solve for the remaining parts.

EX 3

Think back to your congruence postulates in Geometry, ASA, AAS, SAS, SSS and identify each problem above with its postulate.

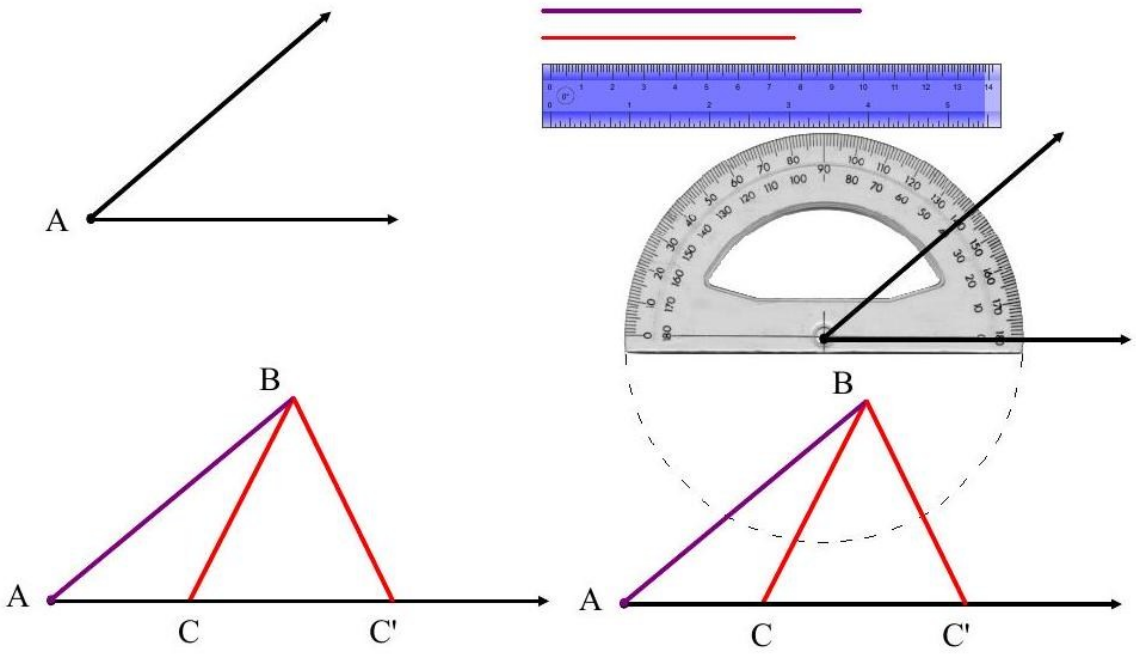
Let's address the dreaded SSA postulate.

EX 4

If $\sin(\alpha) = 0.5$ in triangle ABC, what is the measure of the angle at vertex A?

Ambiguous Case: Here is an example that leads to two different triangles in the case of SSA.

Given $\triangle$ ABC with $\alpha = 40°, c = 10$ cm, and $a = 8$ cm, solve for the other parts.



More Ambiguity

EX 5

In the previous example, consider each of these.

5a)

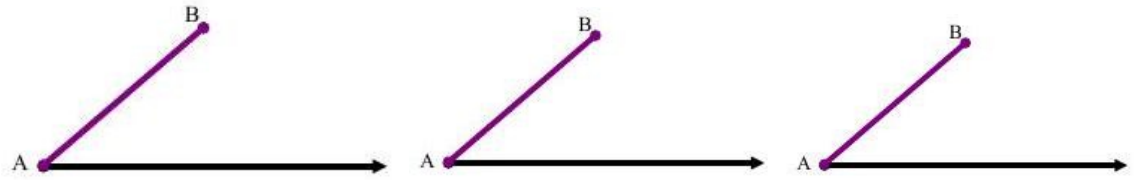
What if $a = 2$ cm ?

5b)

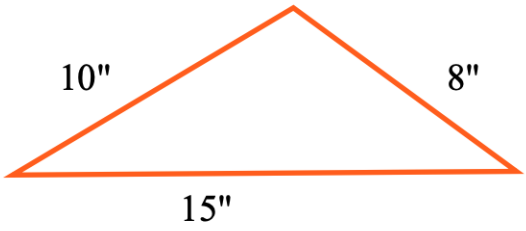
Is there a value for a which produces exactly one triangle?

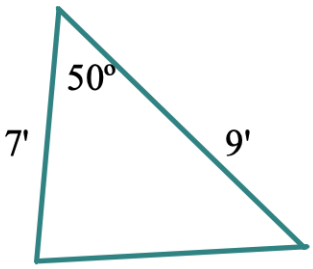
5c)

What if $a = 10$ cm ? produces exactly one triangle?



Now think about the other two postulates, SSS and SAS . Can we use the Law of Sines to solve for parts on these?





It becomes necessary to have another law.

** The app used in this lesson is at this link:
https://www.geogebra.org/m/CvtkyRM5