

Math 1060 ~ Trigonometry

26 Conic Sections: Parabolas

Learning Objectives

In this section you will:

- Learn the definition of a parabola in a plane.
- Know and use the standard equation of a parabola.
- Sketch the graph of a parabola.
- Determine the vertex, focus and directrix of a parabola.

$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2 \sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

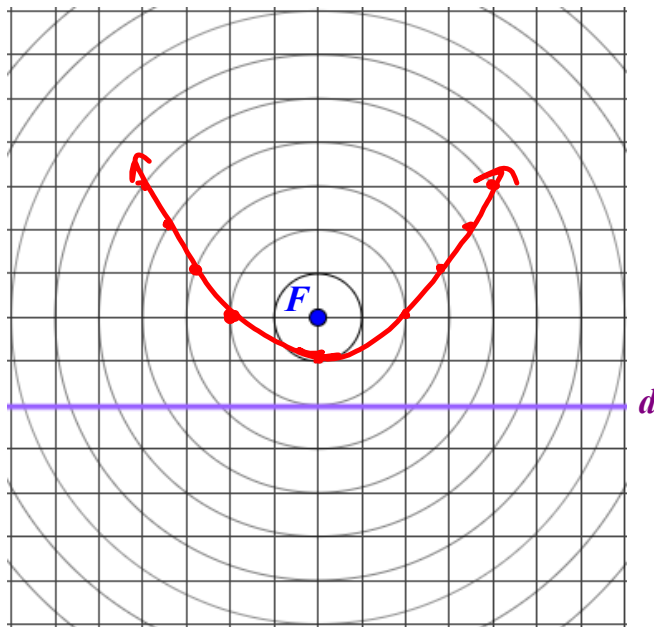
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Parabolas

A parabola is the set of points of equal distance from a given point and a given line.

The point is called the focus and the line is called the directrix.

Ex 1: Plot several points which are equidistant from F , the focus and d , the directrix.



Vocabulary for Parabolas

Focus **F**

Directrix **d**

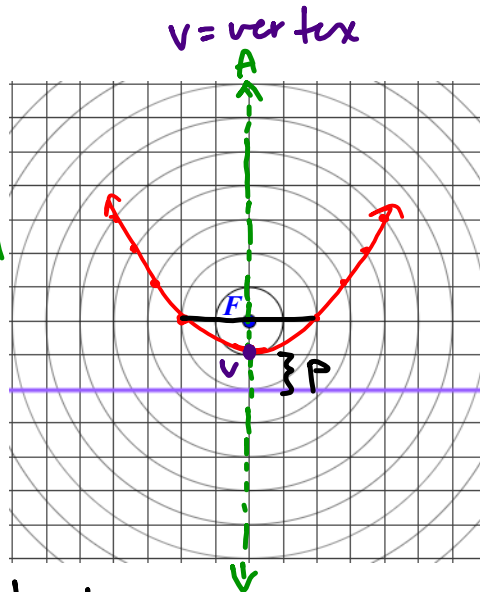
Axis of symmetry - line \perp to **d**
goes thru focus

Latus rectum (focal chord)

line segment \parallel
to **d**, thru **F**,

Connecting parabola

length of focal chord = $|4p|$



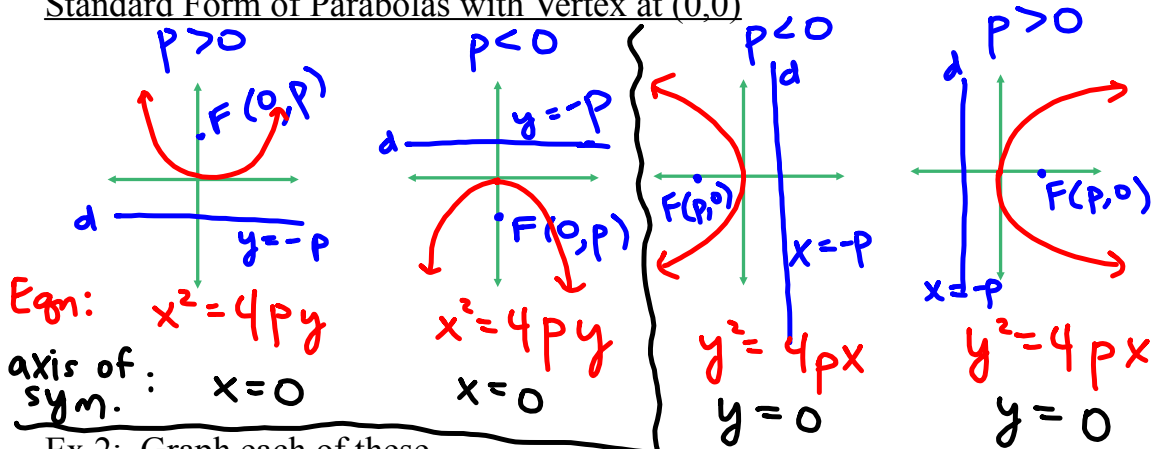
p = dist.
from
vertex
 d to
 d (also
= dist.
from
vertex
to F)

We can develop a formula for the parabola.

Let (x,y) be any point on the parabola.

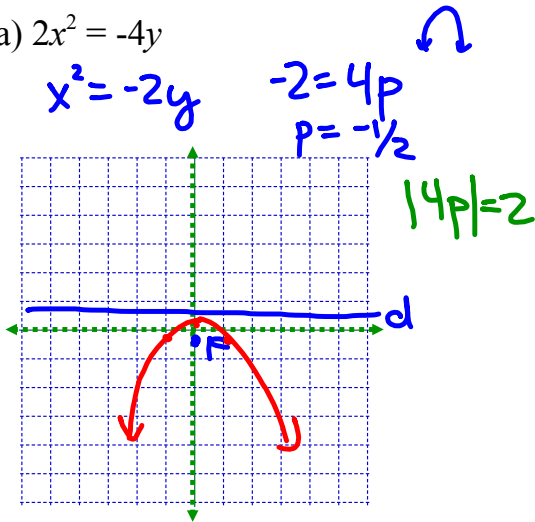
Let F be at $(0,c)$ and the line L be at $y = -c$.

Standard Form of Parabolas with Vertex at (0,0)

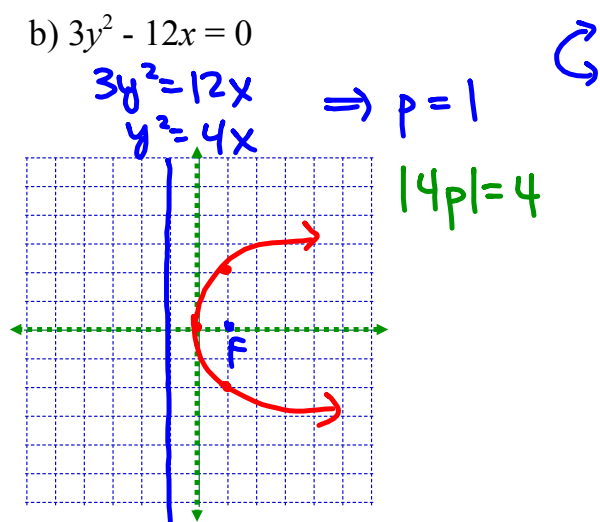


Ex 2: Graph each of these.

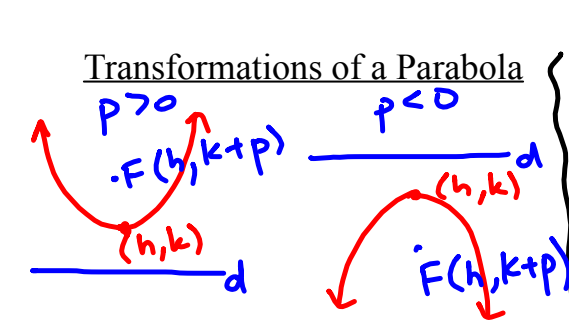
a) $2x^2 = -4y$



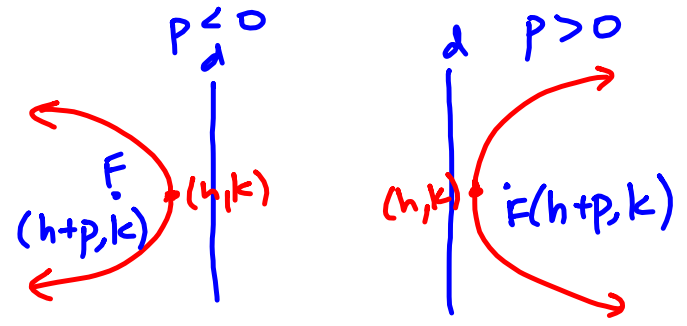
b) $3y^2 - 12x = 0$



Transformations of a Parabola



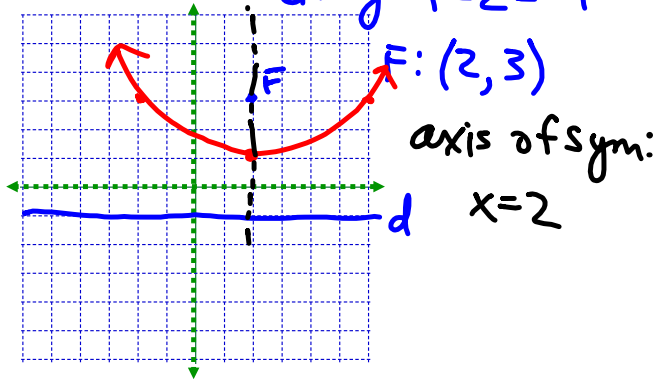
Eqn: $(x-h)^2 = 4p(y-k)$
 d: $y = k-p$
 axis of sym.: $x = h$



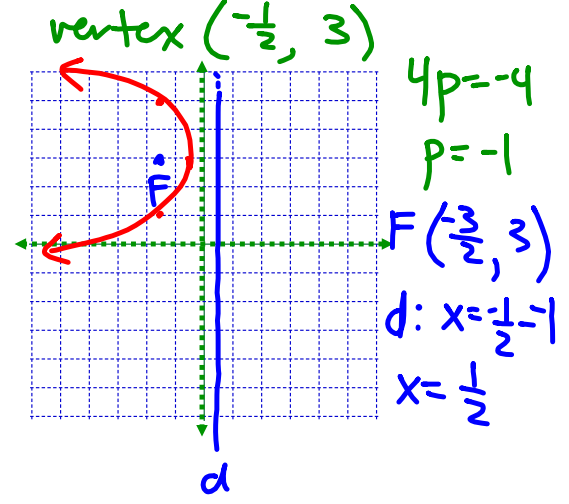
$(y-k)^2 = 4p(x-h)$
 d: $x = h-p$
 $y = k$

Ex 3: Graph each of these. You may need to complete the square on one of them to put it in standard form.

a) $(x-2)^2 = 8(y-1)$ vertex $(2, 1)$
 $4p = 8 \Leftrightarrow p = 2$
 d: $y = 1 - 2 = -1$



b) $y^2 - 6y = -4x - 11$
 $y^2 - 6y + 9 = -4x - 11 + 9$
 $(y-3)^2 = -4(x-2)$
 $(y-3)^2 = -4(x + \frac{1}{2})$



Ex 4: Find the equation of a parabola with the given information.

a) directrix at $y = -4$, vertex at $(4, -1)$

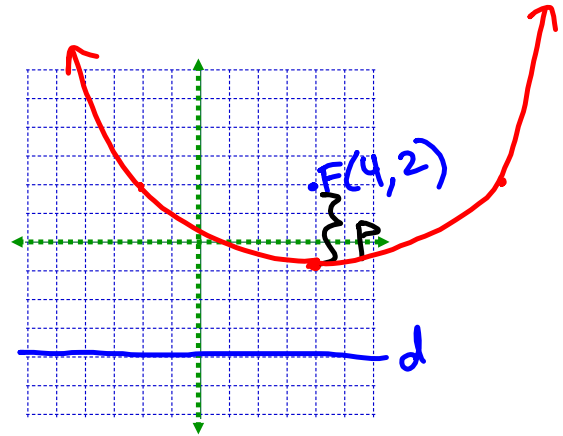
concave up parabola

$$(x-h)^2 = 4p(y-k)$$

$$(x-4)^2 = 4p(y+1)$$

$$(x-4)^2 = 12(y+1)$$

$$p = 3$$



b) vertex at $(4, 2)$, passing through $(-3, -4)$ with axis parallel to the x-axis.

either concave down or left, but axis \parallel to x-axis
 \Rightarrow parabola is concave left

$$(y-k)^2 = 4p(x-h), \quad p < 0$$

$$(y-2)^2 = 4p(x-4)$$

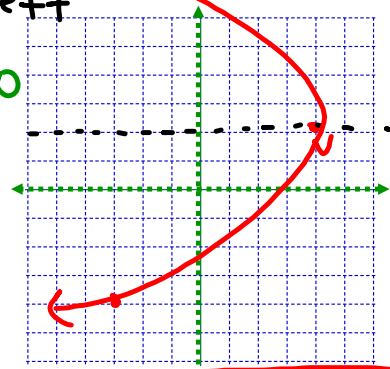
plug in 2nd pt:

$$(-4-2)^2 = 4p(-3-4)$$

$$36 = 4p(-7)$$

$$9 = -7p$$

$$p = -\frac{9}{7}$$



$$(y-2)^2 = -\frac{36}{7}(x-4)$$