

Math 1060 ~ Trigonometry

4 The Six Trigonometric Functions

The Trigonometric Functions

In addition to the sine and cosine functions, there are four more.

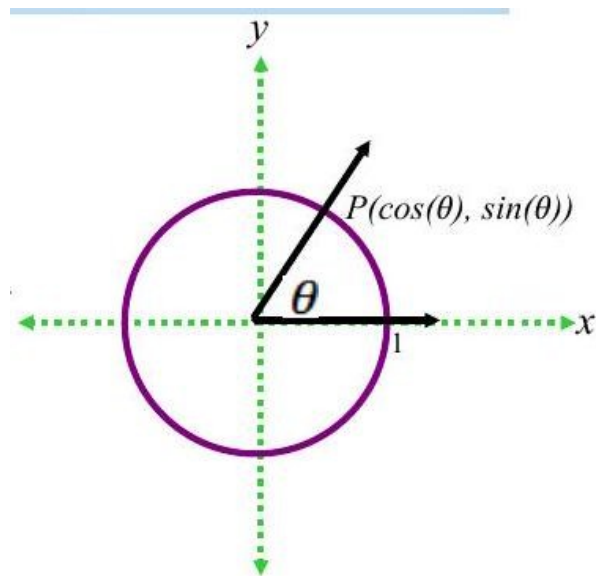
Trigonometric Functions: Suppose θ is an angle plotted in standard position and $P(x, y)$ is the point on the terminal side of θ which lies on the Unit Circle. The circular functions are defined as follows.

- The sine of θ , denoted $\sin(\theta)$, is defined by $\sin(\theta) = y$.
- The cosine of θ , denoted $\cos(\theta)$, is defined by $\cos(\theta) = x$.
- The tangent of θ , denoted $\tan(\theta)$, is defined by $\tan(\theta) = \frac{y}{x}$, provided $x \neq 0$.
- The cosecant of θ , denoted $\csc(\theta)$, is defined by $\csc(\theta) = \frac{1}{y}$, provided $y \neq 0$.
- The secant of θ , denoted $\sec(\theta)$, is defined by $\sec(\theta) = \frac{1}{x}$, provided $x \neq 0$.
- The cotangent of θ , denoted $\cot(\theta)$, is defined by $\cot(\theta) = \frac{x}{y}$, provided $y \neq 0$.

EX 1

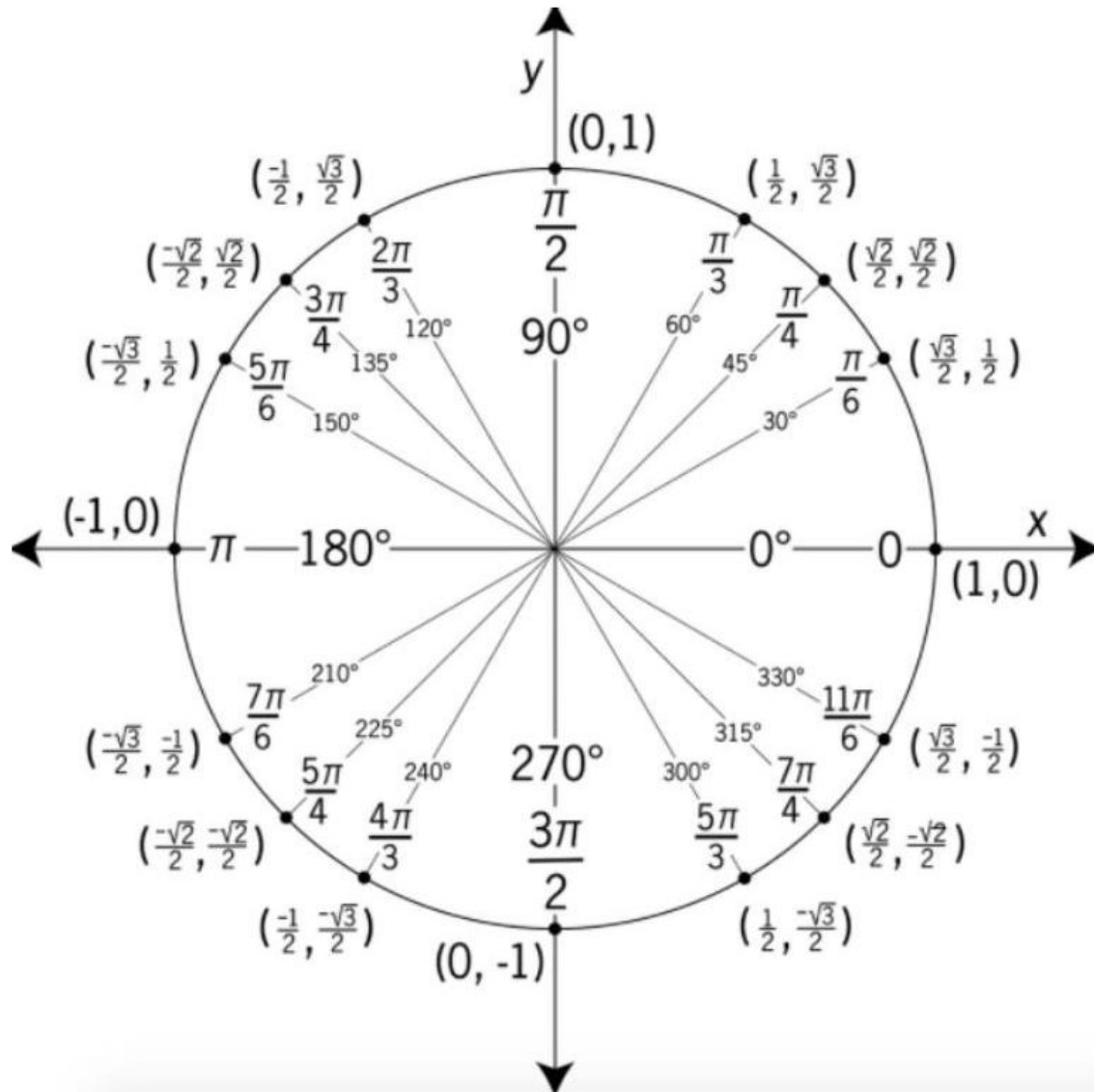
Assume θ is $\frac{\pi}{3}$ in this picture.

Find the six trigonometric functions of θ .



EX 2

Determine the tangent values for the first quadrant and each of the quadrant angles on this Unit Circle.



Reciprocal and Quotient Identities

- $\tan(\theta) = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)}$, provided $\cos(\theta) \neq 0$; if $\cos(\theta) = 0$ then $\tan(\theta)$ is undefined.
- $\cot(\theta) = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)}$, provided $\sin(\theta) \neq 0$; if $\sin(\theta) = 0$ then $\cot(\theta)$ is undefined.
- $\sec(\theta) = \frac{1}{x} = \frac{1}{\cos(\theta)}$, provided $\cos(\theta) \neq 0$; if $\cos(\theta) = 0$ then $\sec(\theta)$ is undefined.
- $\csc(\theta) = \frac{1}{y} = \frac{1}{\sin(\theta)}$, provided $\sin(\theta) \neq 0$; if $\sin(\theta) = 0$ then $\csc(\theta)$ is undefined.

EX 3

Find the indicated value, if it exists.

3a)

$$\sec 30^\circ$$

3b)

$$\csc \frac{11\pi}{6}$$

3c)

$$\cot(2)$$

3d)

$\tan \theta$, where θ is any angle coterminal with 270° .

3e)

$\cos \theta$, where $\csc \theta = -2$ and $\frac{3\pi}{2} < \theta < 2\pi$.

3f)

$\sin \theta$, where $\tan \theta = \sqrt{3}$ and θ is in Q III.

Generalized Reference Angle Theorem

The values of the trigonometric functions of an angle, if they exist, are the same, up to a sign, as the corresponding trigonometric functions of the reference angle.

More specifically, if α is the reference angle for θ , then $\cos \theta = \pm \cos \alpha$, $\sin \theta = \pm \sin \alpha$. The sign, + or -, is determined by the quadrant in which the terminal side of θ lies.

EX 4

Determine the reference angle for each of these. Then state the cosine and sine and tangent of each.

4a)

$$-225^\circ$$

4b)

$$\frac{11\pi}{6}$$

4c)

$$-\frac{3\pi}{4}$$

Finding Angles that Satisfy Cosine and Sine Equations

EX 5

Find all of the angles on the unit circle which satisfy the given equation.

5a)

$$\sin \theta = 0$$

5b)

$$\cos \theta = -\frac{1}{2}$$

5c)

$$\sin \theta = \frac{\sqrt{2}}{2}$$

Finding Angles that Satisfy Other Trigonometric Equations

EX 6

Find all of the angles on the unit circle which satisfy the given equation.

6a)

$$\tan \theta = -1$$

6b)

$$\sec \theta = 2$$

6c)

$$\cot \theta = 0$$