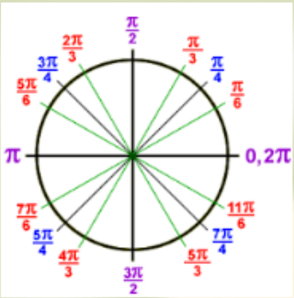
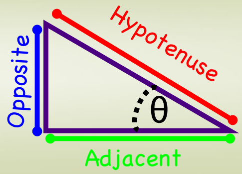
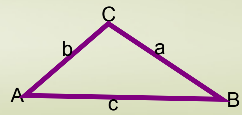
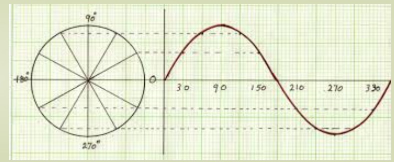








$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2 \sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Math 1060 ~ Trigonometry

7 Graphing The Cosine and Sine Functions

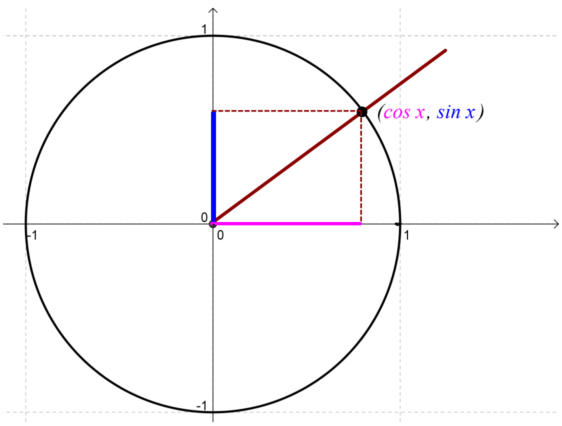
Learning Objectives

In this section you will:

- **Graph the cosine and sine functions.**
- **Learn the properties of the cosine and sine functions, including domain and range, period, phase shift, amplitude and vertical shift.**
- **Identify cosine and sine functions as periodic functions.**
- **Determine whether a periodic function is even or odd.**
- **Use properties to graph periodic functions.**
- **Write an equation from the graph of a sine or cosine function.**

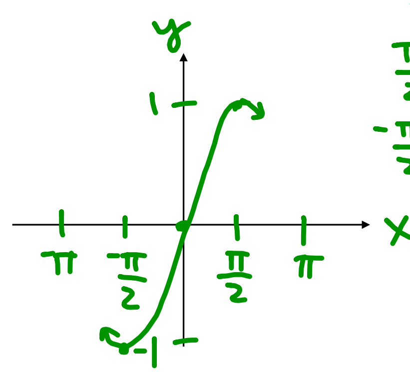
$f(x) = \sin x$

http://tube.geogebra.org/student/m45354?mobile=true



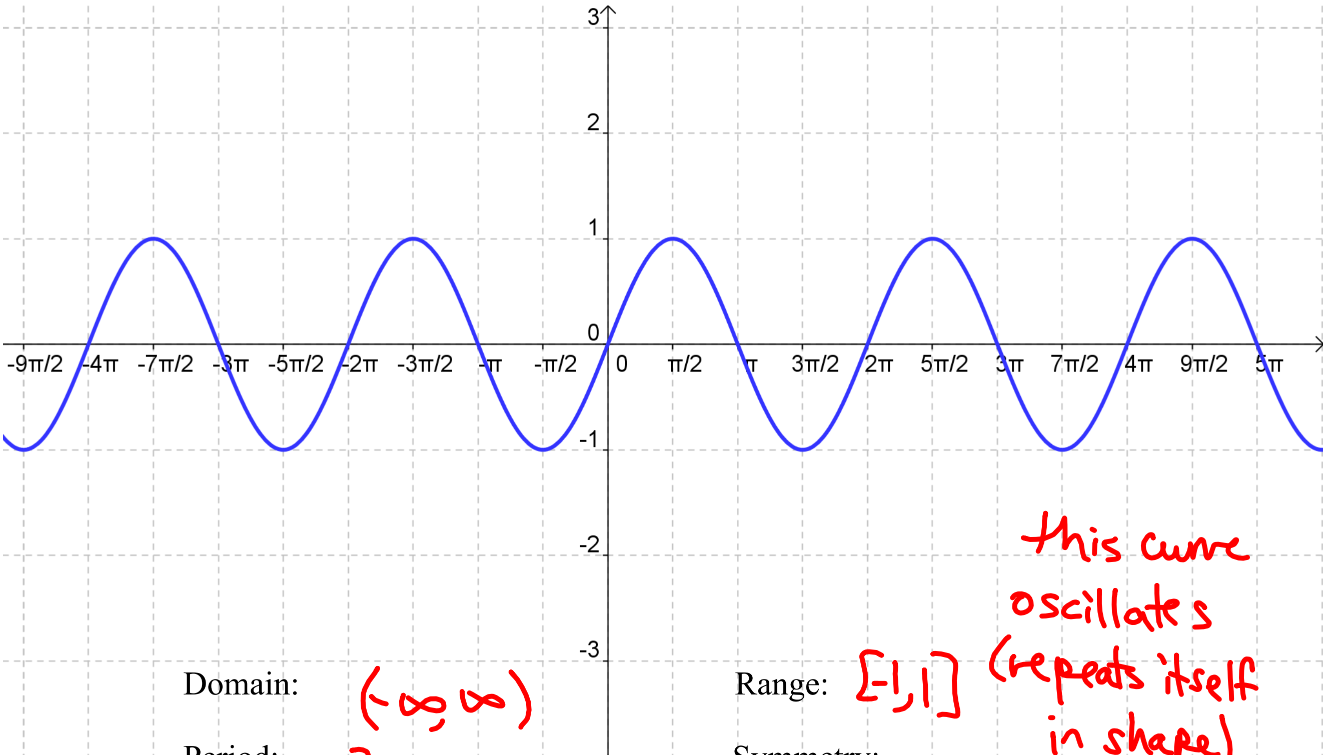
$-1 \le \sin x \le 1$

$y = \sin x$



x	$y = \sin x$
0	0
$\frac{\pi}{2}$	1
$-\frac{\pi}{2}$	-1

Graph of $f(x) = \sin x$



this curve oscillates (repeats itself in shape)

Domain: $(-\infty, \infty)$

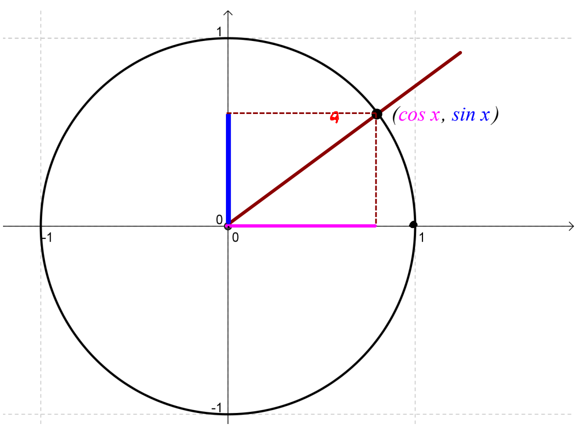
Range: $[-1, 1]$

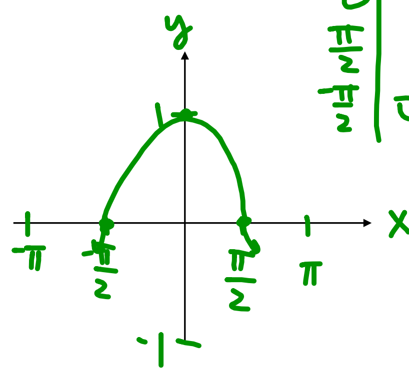
Period: 2π
(horizontal length until shape repeats itself)

Symmetry: wrt origin. (odd fn)

$f(x) = \cos x$

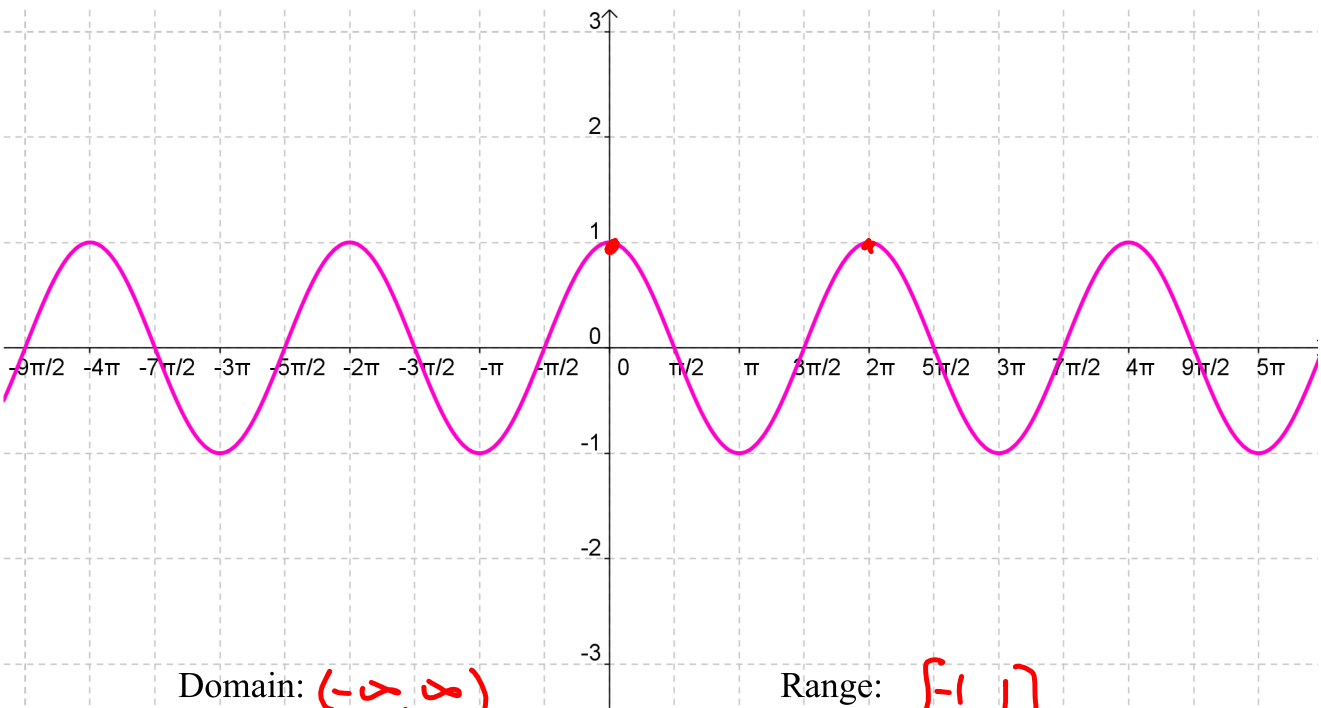
http://tube.geogebra.org/student/m45354?mobile=true





x	$\cos x$
0	1
$\frac{\pi}{2}$	0
$-\frac{\pi}{2}$	0

Graph of $f(x) = \cos x$



Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Period: 2π

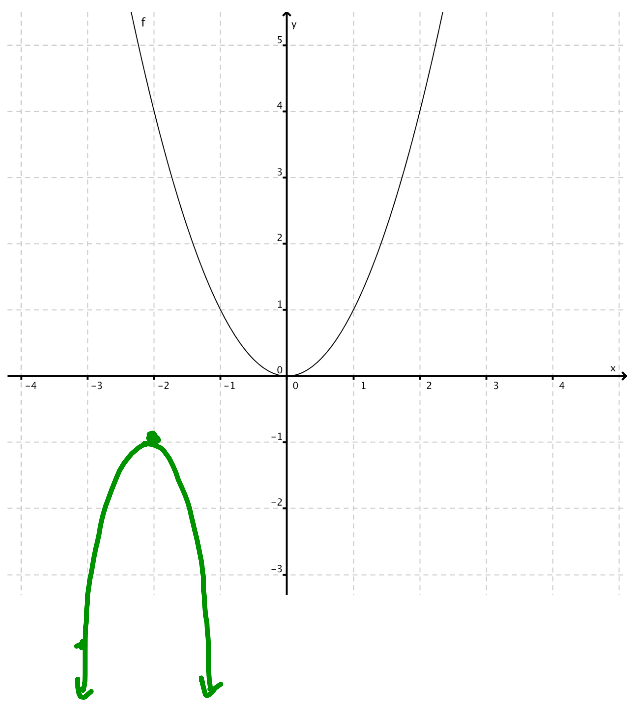
Symmetry: wrt y-axis (even fn)

How can you graph $y = 2\sin(x - \frac{\pi}{3}) + 1$?

This is a transformation of the basic $y = \sin x$ curve.

It may help to remember transformations to one of the algebraic functions.

How does the graph of $y = -3(x+2)^2 - 1$ relate to the graph of $y = x^2$?



vert. reflection

$y = -3(x+2)^2 - 1$

vert. stretch by factor of 3

vert. shift down 1

horiz. shift left 2

In general, remember the effect of a, h and k on the graph of $y = x^2$.
$y = a(x-h)^2 + k$

(h,k) new vertex $\left(\begin{array}{l} h = \text{horiz. shift} \\ k = \text{vert. shift} \end{array}\right)$

$|a|$ = vert. "stretch" factor $\left(\begin{array}{l} \text{if } |a|>1, \text{ stretch} \\ \text{if } |a|<1, \text{ shrink} \end{array}\right)$

$\left\{\begin{array}{l} \text{if } a>0, \text{ no vert. reflection (concave up)} \\ \text{if } a<0, \text{ vert. reflection (concave down)} \end{array}\right.$

$y = A\sin(b(x-h))+k$

What effect do A, *b, h* and k have on the graph of trigonometric functions?

Let's look at it one part at a time: $\boxed{y = A \sin x}$

multiplying by A (on the outside of the fn) causes a vertical stretch/shrink

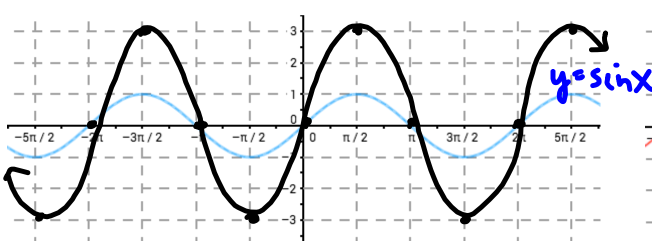
- Amplitude: $|A|$

$y=\sin x$, $y=\cos x$
A=1

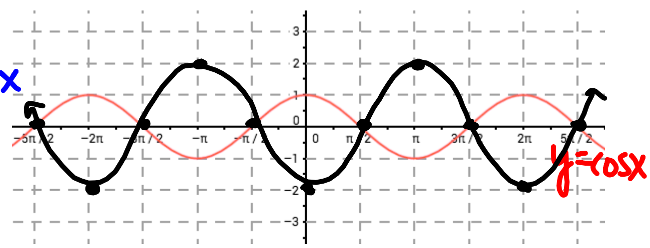
amplitude = max distance (vertically) traveled from the horizontal axis of oscillation; it's half the distance from highest y-value to lowest y-value.

Ex 1: Graph each of these.

$y = 3\sin x$ A=3



$y = -2\cos x$ A=2



vert. stretch by factor of 2

and vert. reflection

Periodic Functions

A function is periodic if there is a real number p so that $f(x+p) = f(x)$. The smallest positive number p, if it exists is called the period of f.

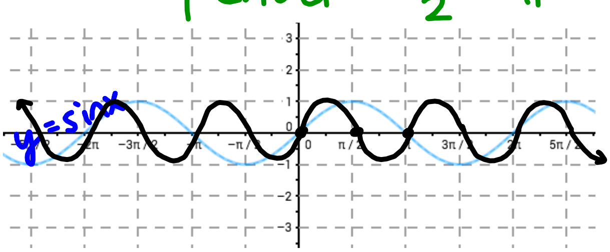
$\boxed{y = \sin(bx)}$ $\Rightarrow$ period $= 2\pi\left(\frac{1}{b}\right) = \frac{2\pi}{b}$

- Period = horiz. distance before graph repeats itself. (normally for $y = \sin x$ and $y = \cos x$ period $= 2\pi$)

Ex 2: Graph each of these.

$y = \sin(2x)$

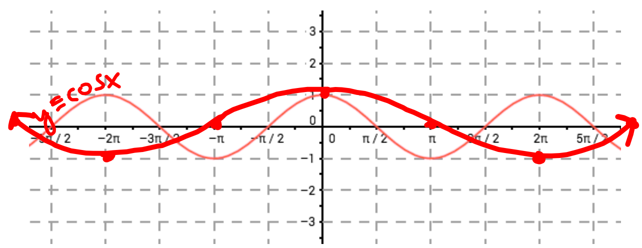
period $= \frac{2\pi}{2} = \pi$



amplitude = 1 (0, 0)

period $= \frac{2\pi}{1/2} = 4\pi$

$y = \cos(\frac{1}{2}x)$



amplitude = 1 (0, 1)

$\boxed{y = \sin(x-h)}$

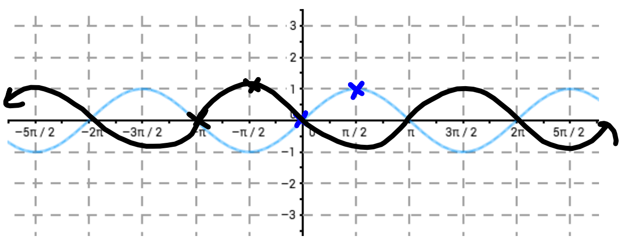
note: $x-h=0$
$x=h$

- Horizontal shift (phase shift) = h

Ex 3: Graph each of these.

$y = \sin(x+\pi)$

horiz. shift $= -\pi$



amp. = 1
period = 2π

Note: this is same as $y = -\sin x$

shift (rt $\frac{\pi}{2}$)

$y = \cos(x - \frac{\pi}{2})$

horiz. shift $= \frac{\pi}{2}$

amp = 1
period = 2π

Note: this is same as $y = \sin x$

$y = \sin(b(x - h))$

- Period = $\frac{2\pi}{b}$
- Horizontal shift = h

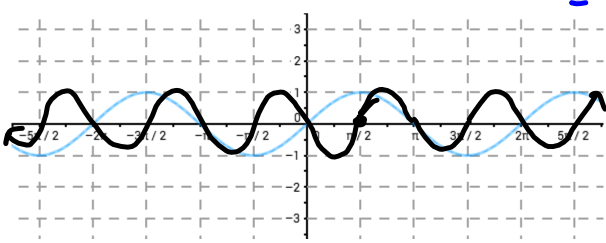
WARNING: must be in form $y=\sin(b(x-h))$ to decide horiz shift.

Ex 4: Graph each of these.

$y = \sin(2x - \pi)$

$y=\sin\left(2\left(x-\frac{\pi}{2}\right)\right)$

period $= \frac{2\pi}{2} = \pi$, horiz. shift $= \frac{\pi}{2}$

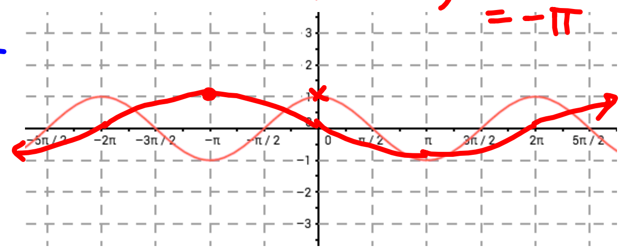


amp = 1

$y = \cos\left(\left(\frac{1}{2}\right) x + \frac{\pi}{2}\right)$

$y=\cos\left(\frac{1}{2}(x+\pi)\right)$

period $= \frac{2\pi}{1/2} = 4\pi$, horiz. shift $= -\pi$



amp = 1

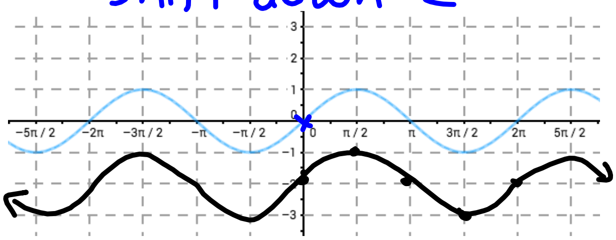
$$\boxed{y = \sin(x) + k}$$

Vertical Shift = k

Ex 5: Graph each of these.

$y = \sin x - 2$

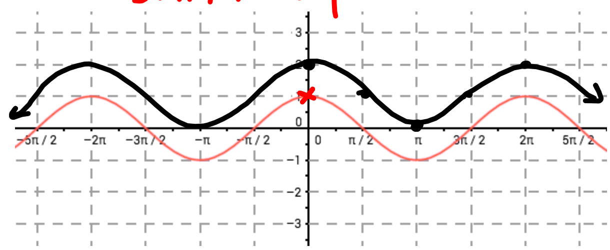
shift down 2



amp = 1
period = 2π

$y = \cos x + 1$

shift up 1



amp = 1
period = 2π

So, when we graph a sine or cosine function there are these things to consider:

- Amplitude
- Period
- Phase shift (horizontal)
- Vertical shift

Ex 6: List the transformations of this function.

$y = 3\cos(2x - \pi) + 1$ $= 3\cos\left(2\left(x - \frac{\pi}{2}\right)\right) + 1$

Amplitude 3 ✓

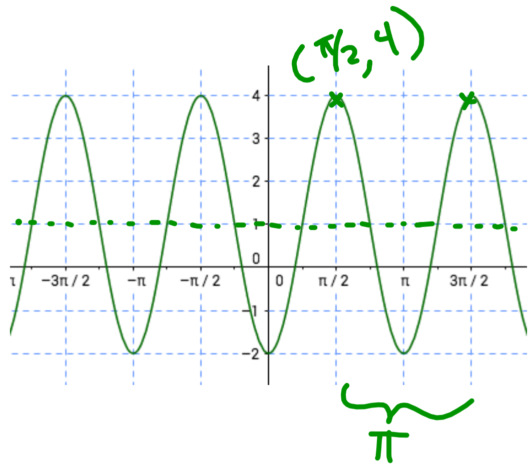
Period $\frac{2\pi}{2} = \pi$ ✓

Phase shift (horizontal)

$\frac{\pi}{2}$ (right) ✓

Vertical shift

1 (up) ✓



Ex 7: List the transformations of this function. $f(x) = -2\sin(4x - \pi) - 2$.

$= -2\sin\left(4\left(x - \frac{\pi}{4}\right)\right) - 2$

Amplitude

$|-2| = 2$

Period

$\frac{2\pi}{4} = \frac{\pi}{2}$

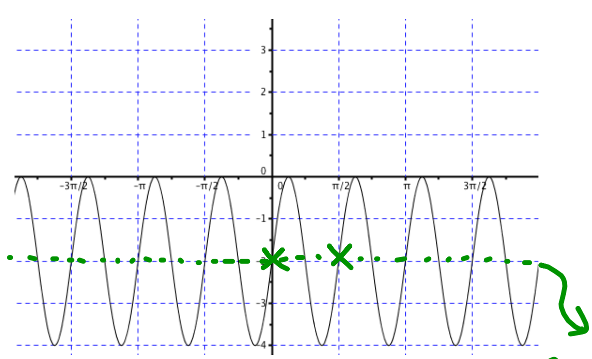
★ Phase shift (horizontal)

$\frac{\pi}{4}$ (right)

Vertical shift

-2 (down)

★ reflection: vertical



axis of oscillation

Ex 8: Analyze the transformations and write a function equation of this graph using the ① cosine function and then one using the ② sine function.

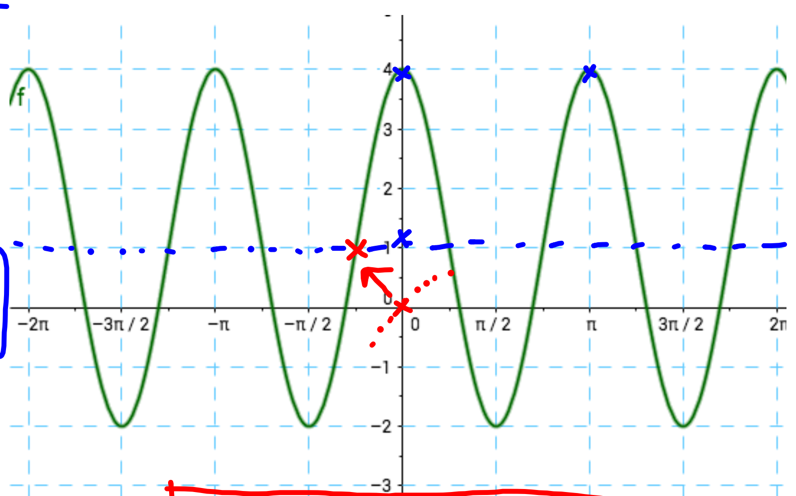
① Period: $\pi = \frac{2\pi}{2}$

Amplitude: 3

Horizontal shift: 0

Vertical shift: 1

$\boxed{y = 3\cos(2x) + 1}$



② period = π
amp = 3
vertical shift = 1
horiz. shift = $-\frac{\pi}{4}$

$x = -\frac{\pi}{4} \Leftrightarrow x + \frac{\pi}{4} = 0$

$\boxed{y = 3\sin\left(2\left(x + \frac{\pi}{4}\right)\right) + 1}$

Here are some applets in case you want to play with the transformation variables.

http://www.analyzemath.com/trigonometry/sine.htm

http://tube.geogebra.org/student/m45354?mobile=true

Here are instructions and the equation format from the text for graphing a periodic (sinusoidal) function.

For $\omega > 0$, the functions

$$C(x) = A\cos(\omega x + \phi) + B \text{ and } S(x) = A\sin(\omega x + \phi) + B$$

- have period $\dfrac{2\pi}{\omega}$
- have amplitude $|A|$
- have phase shift $-\dfrac{\phi}{\omega}$
- have vertical shift B