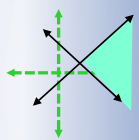
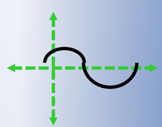
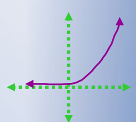
$5x-2y \leq 75$



$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$S = Pe^{rt}$



$APY = (1+\frac{r}{n})^n - 1$

Math 1090 ~ Business Algebra

Section 3.2 Parabolas: Quadratic Equations in Two Variables

Objectives:

- Identify a quadratic function, including the dependent and independent variables.
- Sketch a graph of a quadratic function.
- Identify the vertex, the axis of symmetry, concavity, y-intercept and roots of a quadratic function.

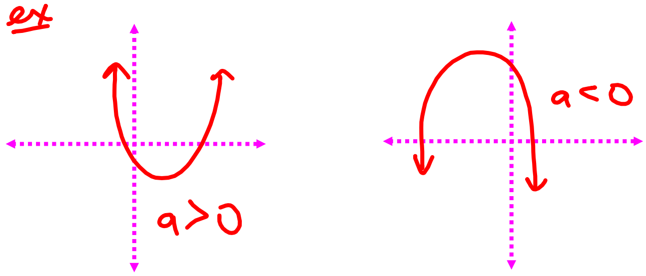
A quadratic function in two variables can be written in the form

$\boxed{y = f(x) = ax^2 + bx + c}$ $\underline{a \neq 0}$, $a,b,c \in \mathbb{R}$

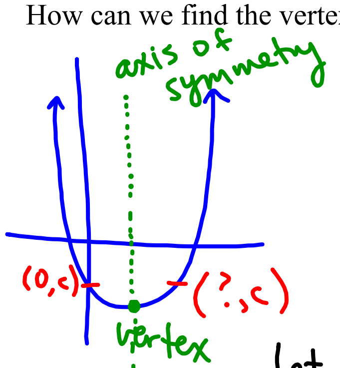
a,b,c constants

x is the independent variable

ex



How can we find the vertex?



$? = \frac{-b}{a}$

$y = ax^2 + bx + c$

check: plug in $x=0$

$\Rightarrow y = a(0) + b(0) + c = c$

so parabola goes through pt $(0, c)$.

let $y = c$. We get

$c = ax^2 + bx + c$

$0 = ax^2 + bx$

$0 = x(ax+b)$

$x = 0$ ✓ or $ax + b = 0$

$ax = -b$

$x = \frac{-b}{a}$

$\Rightarrow$ vertex is halfway between $x=0$ and $x = \frac{-b}{a}$

i.e. vertex occurs when

$$x = \frac{1}{2}\left(\frac{-b}{a}\right) = \frac{-b}{2a}$$

vertex: $\boxed{\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)}$

$y = ax^2 + bx + c$

Ex 1: For $y = -2x^2 - 4x + 6$ (parabola, a.k.a. quadratic fn)

a) Find the vertex.

$a = -2, \; b = -4$

$x = \frac{-b}{2a} = \frac{-(-4)}{2(-2)} = -1$

vertex: $\boxed{(-1, 8)}$

$y = -2(-1)^2 - 4(-1) + 6 = -2 + 4 + 6 = 8$

b) Is the vertex a min or max point?

$a = -2 < 0 \Rightarrow$ parabola is concave down

$\Rightarrow$ vertex is $\boxed{\text{max pt}}$

Ex 2: For $y = x^2 - 6x + 9$, $a=1,\ b=-6,\ c=9$

a) Find the vertex. $(3,0)$

$x = \frac{-b}{2a} = \frac{6}{2} = 3$ $y = 3^2 - 6(3) + 9$

$y = 0$

b) Is it a min or max point?

$a = 1 > 0$ vertex is min

c) Find the zeros/roots of the graph.

$0 = x^2 - 6x + 9$

$0 = (x-3)(x-3)$

$x - 3 = 0 \iff \boxed{x=3}$

d) Find the axis of symmetry

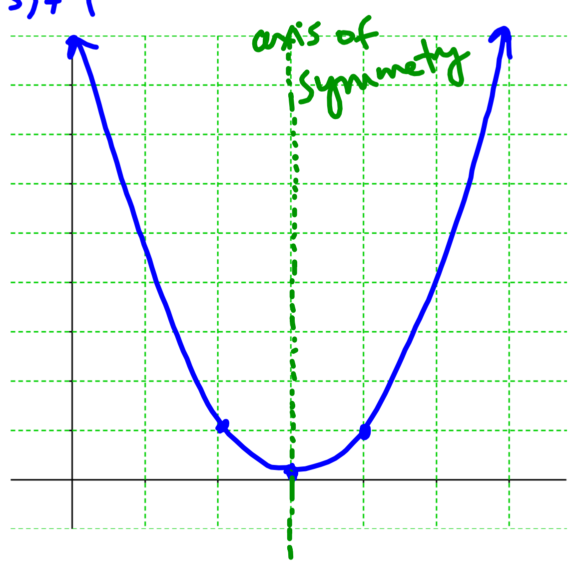
$\boxed{x=3}$

e) Find the y-intercept.

$y = 0^2 - 6(0) + 9 = 9$

$\boxed{(0,9)}$

f) Sketch the graph



Ex 3: For $y = -x^2 + 4x + 5$, $a=-1$, $b=4$, $c=5$

a) Find the vertex.

$x = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2$ $y = -(2^2) + 4(2) + 5 = -4 + 8 + 5$
$= 9$

$\boxed{(2,9)}$

b) Is this parabola concave up or concave down?

$a = -1 < 0 \Rightarrow$ $\boxed{\text{concave down}}$

c) Find the x and y-intercepts of the graph.

x-int: $(-1, 0)$ $(5, 0)$

$(0 = -x^2 + 4x + 5)(-1)$

$0 = x^2 - 4x - 5$

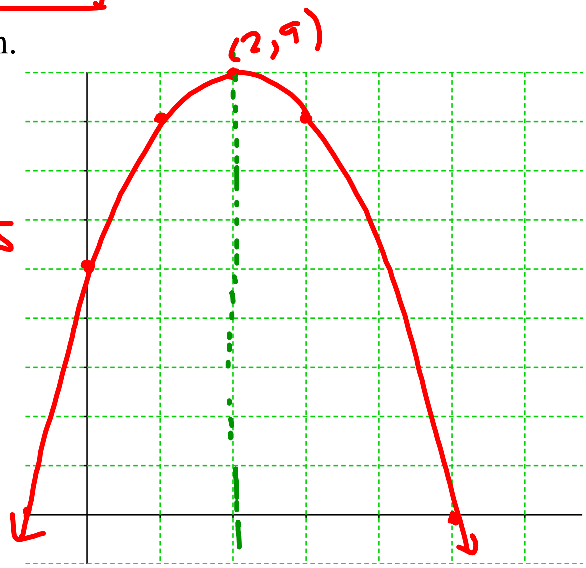
$0 = (x-5)(x+1)$

$x = 5, -1$

y-int: $(0, 5)$

$y = 0 + 0 + 5$

$y = 5$



d) Find the axis of symmetry

$x = 2$

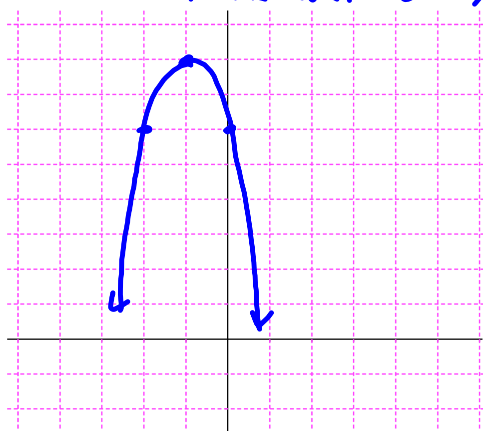
e) Sketch the graph

Ex 4: For the parabola from example 1, $y = -2x^2 - 4x + 6$, sketch the graph.

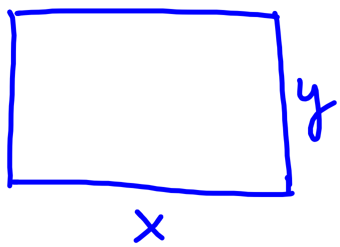
coefficient of x^2 is -2

vertex $(-1, 8)$

concave down



Ex 5: If 100 ft of fencing is used to enclose a rectangular yard, find the area function. Find the dimensions of the rectangle that maximizes the area.



$P = 100\text{ ft} = 2x + 2y$

$\Leftrightarrow \quad 100 = 2x + 2y$

$50 = x + y$

$y = 50 - x$

$A = A(x) = xy$

$A = x(50 - x)$

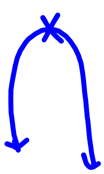
$A(x) = 50x - x^2$

$A(x) = -x^2 + 50x$

$\Rightarrow$ area is a quadratic fn of x.

and leading coefficient is negative

$\Rightarrow$ we have concave down parabola



$\Rightarrow$ max area occurs at vertex

vertex: at $x = \frac{-b}{2a} = \frac{-50}{2(-1)} = 25$

dimensns of rectangle: $x = 25$, $y = 50 - 25 = 25$

$\boxed{25\text{ ft} \times 25\text{ ft}}$