$5x - 2y \le 75$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$S = Pe^{rt}$

$APY = (1 + \frac{r}{n})^{n} - 1$

Math 1090 ~ Business Algebra

Section 3.3 Quadratic Business Applications

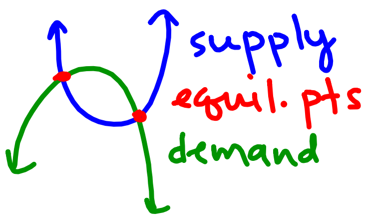
Objectives:

- Set up and solve quadratic equations as they apply to business situations.

Quadratic Business Applications

Supply, Demand and Market Equilibrium

Ex 1: If the supply function for a commodity is $p = q^2 + 8q + 20$ and the demand function is $p = 100 - 4q - q^2$, find the equilibrium quantity and the equilibrium price.



S ① $p = q^2 + 8q + 20$

D ② $p = 100 - 4q - q^2$

use substitution:

$q^2 + 8q + 20 = 100 - 4q - q^2$

$\frac{2q^2 + 12q - 80}{2} = \frac{0}{2}$

$q^2 + 6q - 40 = 0$

$(q + 10)(q - 4) = 0$

$q + 10 = 0$ or $q - 4 = 0$

$\cancel{q = -10}$ $\boxed{q = 4}$

equilibrium pt:

$(4, \$68)$

① $p = 4^2 + 8(4) + 20$

$= 16 + 32 + 20$

$= 68$

Ex 2: For the last example, if an \$8.00 tax is placed on production and passed through the supplier, find the new equilibrium point.

supply: (1) $p = q^2 + 8q + 20 + 8$

demand: (2) $p = 100 - 4q - q^2$

$$q^2 + 8q + 28 = 100 - 4q - q^2$$

$$2q^2 + 12q - 72 = 0$$

$$2(q^2 + 6q - 36) = 0$$

$$q^2 + 6q - 36 = 0 \qquad \text{(not factorable)}$$

$$q = \frac{-6 \pm \sqrt{6^2 - 4(1)(-36)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 + 4(36)}}{2} = \frac{-6 \pm \sqrt{36(5)}}{2}$$

$$= \frac{-6 \pm 6\sqrt{5}}{2} = \frac{\cancel{2}(-3 \pm 3\sqrt{5})}{\cancel{2}}$$

$q = -3 \pm 3\sqrt{5}$ note: $-3 - 3\sqrt{5}$ is negative, but $q \geq 0$.

soln: $q = -3 + 3\sqrt{5} \simeq 3.7$

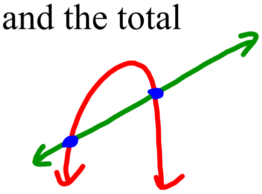
$\Rightarrow$ (1) $p = (-3 + 3\sqrt{5})^2 + 8(-3 + 3\sqrt{5}) + 28$

$\simeq \$71.42$

$\Rightarrow$ equil. pt. $\sim (3.7, \$71.42)$

Break-Even Points and Maximization

Ex 3: If a company has total costs $C(x) = 1600 + 1500x$ and the total revenue is $R(x) = (1600 - x)x$, find the break even points.



① $y = R(x) = -x^2 + 1600x$

② $y = C(x) = 1500x + 1600$

Break even points occur when

$R(x) = C(x) \Leftrightarrow P(x) = 0$

note:
$1600 = 80(20)$

$$-x^2 + 1600x = 1500x + 1600$$

$$0 = x^2 + 1500x - 1600x + 1600$$

$$0 = x^2 - 100x + 1600$$

$$0 = (x - 80)(x - 20)$$

$$0 = x - 80 \quad \text{or} \quad 0 = x - 20$$

$$x = 80 \quad \text{or} \quad x = 20$$

$$y = C(80) = 1600 + 1500(80) = \$121{,}600$$

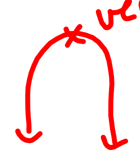
$$y = C(20) = 1600 + 1500(20) = \$31{,}600$$

break-even pts (profit is zero)

(80, \$121,600)
and (20, \$31,600)

Ex 4: Find the maximum revenue given $R(x) = 1600x - x^2$.

vertex is max pt.



$R(x) = -x^2 + 1600x$

vertex: $x = \frac{-b}{2a} = \frac{-1600}{2(-1)} = 800$

$\Rightarrow$ max revenue occurs at $x = 800$

$R(800) = -(800)^2 + 1600(800)$

$= \$640{,}000$

Ex 5: Suppose a company has fixed costs of *$4,320,000* and variable costs of *0.8 x - 4000* dollars per unit, where x = the number of units produced. Suppose further that its selling price is *2000 - 1.2 x* dollars per unit.

a) Find the break even point.

$C(x) = 4{,}320{,}000 + (0.8x - 4000)x$

$C(x) = 0.8x^2 - 4000x + 4{,}320{,}000$

$R(x) = (2000 - 1.2x)x = -1.2x^2 + 2000x$

$P(x) = R(x) - C(x) = -1.2x^2 + 2000x - 0.8x^2 + 4000x - 4320000$

$= -2x^2 + 6000x - 4320000$

$= -2(x^2 - 3000x + 2{,}160{,}000) = 0$

$-2(x - 1800)(x - 1200) = 0 \iff x = 1800,\ x = 1200$

$\cancel{x = 1800}$, $R(1800) = -1.2(1800^2) + 2000(1800) = -288{,}000$

$x = 1200$ $R(1200) = -1.2(1200^2) + 2000(1200) = \$672{,}000$

break even pt: $\boxed{(1200, \$672{,}000)}$

b) Find the maximum revenue.

$R(x) = -1.2x^2 + 2000x$

max at parabola vertex

$x = \frac{-2000}{2(-1.2)} = \frac{2500}{3} \approx 833.33$

max $R\left(\frac{2500}{3}\right) = -1.2\left(\frac{2500}{3}\right)^2 + 2000\left(\frac{2500}{3}\right)$

$\approx \boxed{\$833{,}333.33}$

c) Find the maximum profit and the price that yields it.

max

$P(x) = -2x^2 + 6{,}000x - 4{,}320{,}000$

vertex: $x = \frac{-6000}{2(-2)} = 1500$

max profit $= P(1500) = -2(1500^2) + 6000(1500) - 4{,}320{,}000$

$= \boxed{\$180{,}000}$

selling price $= 2000 - 1.2(1500)$

$= \boxed{\$200}$