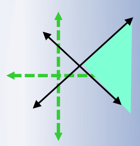
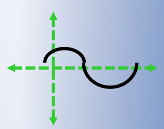
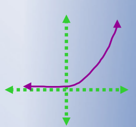
$5x-2y \le 75$



$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$S = Pe^{rt}$



$APY = (1+\frac{r}{n})^n - 1$

Math 1090 ~ Business Algebra

Section 3.5 Rational Functions

Objectives:
- Identify a rational function.
- Determine the domain and intercepts of a rational function.
- Determine vertical and horizontal asymptotes.
- Sketch a rational function.

Definition

Rational Function $f(x)=\frac{n(x)}{d(x)}$

where *n(x)* and *d(x)* are polynomials.

ex ① $f(x)=\frac{3x+5}{x^2-1}$

② $g(x)=\frac{1}{x^3+2x}$

Asymptotes

Vertical asymptotes

ex ① $f(x)=\frac{x}{x-2}$

VA: $x=2$

② $f(x)=\frac{\cancel{(x-1)}(x+3)}{\cancel{(x-1)}(x+2)}$

$=\frac{x+3}{x+2}$, $x\neq 1$

VA: $x=-2$

hole: @ $x=1$ $\left(1,\frac{4}{3}\right)$ $y=\frac{1+3}{1+2}$

Horizontal asymptotes

"end behavior"

① $f(x)=\frac{2x}{x+5}$

as x gets huge $\to \pm\infty$

$f(x)\sim\frac{2x}{x}=2$

$\Rightarrow$ HA: $y=2$

② $g(x)=\frac{x+3}{3x^2+5x-7}$

as x gets huge, $x\to\pm\infty$

we get $g(x)\sim\frac{x}{3x^2}=\frac{1}{3x}\to 0$

$\Rightarrow$ HA: $y=0$

How to graph a rational function

1) find the domain

all allowable x-values

a) find VA

line $x=b$, b is the x-value that makes the den. zero (and num. not zero)

b) find HA

$y=c$, where c is the "end behavior" y-value (i.e. when x is huge)

2) Find x and y-intercepts

x-intercepts: $(d,0)$

y-intercept: $(0,p)$

3) Plot intercept points and at least one point on all sides of the vertical asymptotes.

4) Fill in the graph with smooth curves that approach the asymptotes.

Ex 1: Analyze and graph.

a) $f(x) = \dfrac{2+x}{1-x}$

domain: $x \neq 1$

VA: $x = 1$

HA: $y = -1$

as $x \to \pm\infty$,

$f(x) \sim \dfrac{x}{-x} = -1$

x-intercepts: $(-2, 0)$

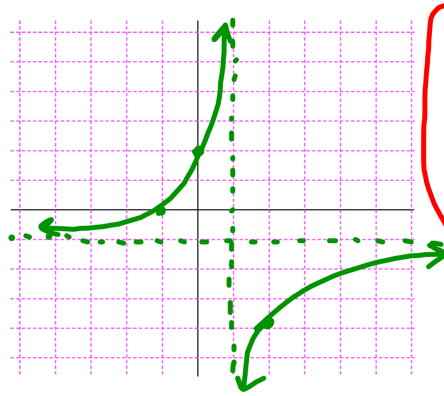
$0 = \dfrac{2+x}{1-x}$

$0 = 2 + x$

$x = -2$

y-intercept: $(0, 2)$

$y = \dfrac{2+0}{1-0} = 2$



rational fn graphs NEVER touch or cross VA

pt on right of VA:

$(2, -4)$ $f(2) = \dfrac{2+2}{1-2} = -4$

b) $f(x) = \dfrac{10}{x^2+2}$

domain: $x \in \mathbb{R}$

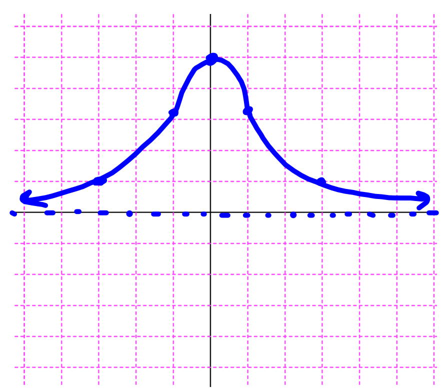
VA: none

HA: $y = 0$

as $x \to \pm\infty$

$f(x) \sim \dfrac{10}{x^2} \to 0$

$(\pm 1, \frac{10}{3})$ $y = \dfrac{10}{1+2}$



x-int: none

$0 = \dfrac{10}{x^2+2}$

$0 \neq 10$

y-int: $(0, 5)$

$y = \dfrac{10}{0+2} = 5$

notice: this fn is even, i.e. if we plug in $\pm x$, we get same y-value

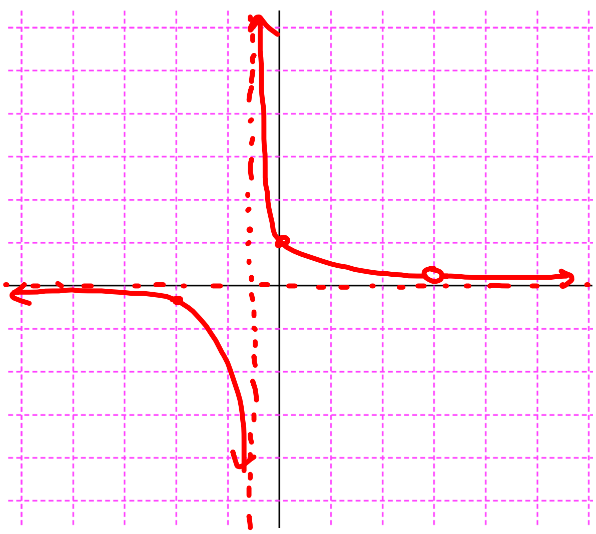
Ex 2: Analyze and graph. $g(x)=\dfrac{x-3}{2x^2-5x-3} = \dfrac{\cancel{x-3}}{(2x+1)\cancel{(x-3)}}$

$y=\dfrac{1}{2x+1}, \; x\neq 3$

another pt: $(-2, \frac{-1}{3})$

$y=\dfrac{1}{2(-2)+1}$

$=\dfrac{-1}{3}$



domain: $x \in \mathbb{R}$, $x \neq 3, \frac{-1}{2}$

VA: $x = \frac{-1}{2}$

hole: $(3, \frac{1}{7})$

$y=\dfrac{1}{2(3)+1}=\dfrac{1}{7}$

x-int: none

$y=\dfrac{1}{2x+1}$

$0=\dfrac{1}{2x+1}$

$0\neq 1$

y-int: $(0,1)$

$y=\dfrac{1}{2(0)+1}$

$=1$

HA: $y=0$

as $x\to\pm\infty$, $y\sim\dfrac{1}{2x}\to 0$