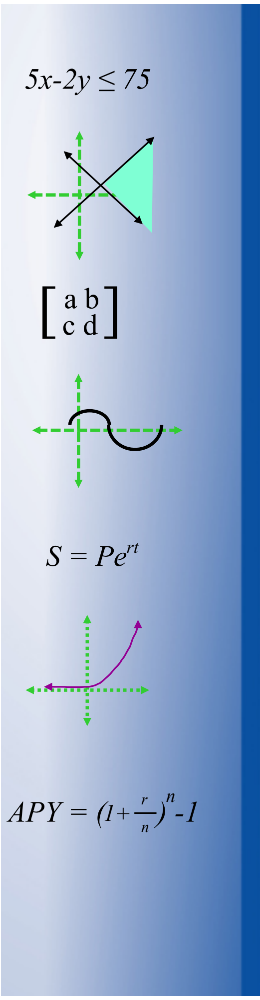

Math 1090 ~ Business Algebra

Section 3.7 Combinations of functions

Objectives:
- Form compositions of two functions.
- Determine the domain of the composite function.
- Perform arithmetic of functions.

Two functions can be combined to for a new function in these ways.

- addition $(f+g)(x)=f(x)+g(x)$
- subtraction $(f-g)(x)=f(x)-g(x)$
- multiplication $(f\cdot g)(x)=f(x)\cdot g(x)$
- division $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$
- composition $(f\circ g)(x)=f(g(x))$ "f of g of x" (nested function)

Ex 1 Given $f(x)=2x+5$ $g(x)=\frac{1}{x^3}$

a) $(f\circ g)(x)$

$=f(g(x))$

"inside out"

$f\left(\frac{1}{x^3}\right)$

$=2\left(\frac{1}{x^3}\right)+5$

"outside in"

$2(g(x))+5$

$=2\left(\frac{1}{x^3}\right)+5$

b) $(f+g)(1)$

$=f(1)+g(1)$

$=(2\cdot 1+5)+\left(\frac{1}{1^3}\right)$

$=7+1=8$

c) $(g\circ f)(1)$

$=g(f(1))$

$g(7)$

$=\frac{1}{7^3}$

$\frac{1}{[f(1)]^3}$

$=\frac{1}{7^3}$

d) $\left(\frac{f}{g}\right)(x)$

$=\frac{f(x)}{g(x)}=\frac{(2x+5)}{\frac{1}{x^3}}\left(\frac{x^3}{x^3}\right)$

$=\frac{2x^4+5x^3}{1}$

$=2x^4+5x^3$

$$f(\heartsuit) = \heartsuit^2 - 1$$

Ex 2: Given $f(x) = x^2 - 1$ $\quad g(x) = \dfrac{x}{2}$ $\quad h(x) = \sqrt{x-1}$, find

a) $(h \circ f)(x)$

$$= h(f(x))$$
$$= h(x^2 - 1)$$
$$= \sqrt{(x^2-1)-1} = \sqrt{x^2 - 2}$$

b) $(g - h)(1)$

$$= g(1) - h(1)$$
$$= \frac{1}{2} - \sqrt{1-1}$$
$$= \frac{1}{2} - 0 = \frac{1}{2}$$

c) $(hf)(3)$

$$= h(3) \cdot f(3)$$
$$= \sqrt{3-1}\,(3^2 - 1)$$
$$= \sqrt{2}(8) = 8\sqrt{2}$$

d) $g(h(x))$

$$= \frac{h(x)}{2}$$
$$= \frac{\sqrt{x-1}}{2}$$

e) $h(f(g(x)))$

$$= h\left(f\left(\frac{x}{2}\right)\right)$$
$$= h\left(\left(\frac{x}{2}\right)^2 - 1\right)$$
$$= h\Big(\underbrace{\frac{x^2}{4} - 1}_{\text{"glob"}}\Big)$$
$$= \sqrt{\left(\frac{x^2}{4} - 1\right) - 1}$$
$$= \sqrt{\frac{x^2}{4} - 2}$$

$$\boxed{\begin{array}{l} f(x) = x^2 - 1 \\ g(x) = \dfrac{x}{2} \\ h(x) = \sqrt{x-1} \end{array}}$$

Ex 3: For these functions, find *g(h(x))* and its domain.

$$g(x)=\frac{5}{x} \qquad h(x)=\sqrt{x-1}$$

domain: $x \neq 0$ $(g(x))$

$x-1 \geq 0 \Leftrightarrow x \geq 1$ $(h(x))$

$g(h(x)) = \frac{5}{h(x)} = \frac{5}{\sqrt{x-1}}$ domain: $\cancel{x \geq 1}$ $x > 1$

Ex 4: The daily cost of producing *x* units in a manufacturing process is *C(x) = 11x + 350*. The number of units produced in *t* hours during a day is given by *x(t) = 10t* for $0 \leq t \leq 8$. Find, simplify and interpret *C(x(t))*.

$$C(x(t)) = C(10t)$$
$$= 11(10t)+350$$
$$= 110t+350$$

notice:

- this is now a fn of t only
- describes cost for a given # of hours worked

ex if we work 5 hrs, cost is $C = 110(5)+350$

$= 550+350 = \$900$