$5x-2y \leq 75$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$S = Pe^{rt}$

$APY = (1+\frac{r}{n})^n - 1$

Math 1090 ~ Business Algebra

Section 4.6 Logarithmic and Exponential Business Applications

Objectives:

- Solve business application problems using logarithmic function strategies.
- Solve business application problems using exponential function strategies.

Ex 1: If \$1000 is invested at 10% compounded continuously, the future value S at any time t (in years) is given by $S = 1000e^{0.1t}$.

a) What is the account worth after one year?

$t = 1$ yr.

$$S = 1000e^{0.1(1)} = 1000e^{0.1} \simeq \$1105.17$$

b) How long will it take for the investment to double?

$S = 2(1000) = 2000$

$t = ?$

$$2000 = 1000e^{0.1t}$$

$$2 = e^{0.1t}$$

$$10 \ln 2 = 0.1t \ (10)$$

$$10 \ln 2 = t$$

$$t \simeq 6.93 \text{ yrs}$$

Ex 2: The population of Mathville grows according to the formula $P = P_0 e^{0.03t}$. If the population was 250,000 in the year 2000, estimate the year in which the population reaches 350,000.

P_0 = initial population (i.e. the population at the time we decide is 0)

let $t=0$, in year 2000

$\Rightarrow P_0 = 250{,}000 \Rightarrow P = 250000e^{0.03t}$

$t = ?$ when $P = 350{,}000$

$$350000 = 250000e^{0.03t}$$

$$\frac{35}{25} = e^{0.03t}$$

$$\ln\left(\frac{35}{25}\right) = 0.03t \rightarrow t = \frac{\ln\left(\frac{35}{25}\right)}{0.03} \simeq 11.2 \text{ yrs.}$$

$\Rightarrow$ in year $\boxed{2011}$

Ex 3: Radioactive Iodine-(3) has a half-life of 8 days. How long does it take to reduce an initial amount of Iodine-(3) to 1% of the initial amount.

$$P = P_0 e^{rt}$$

half-life = 8 days: $t = 8$ days for $P = \frac{1}{2}P_0$

$$\frac{1}{2}\cancel{P_0} = \cancel{P_0}e^{r(8)}$$

$$\frac{1}{2} = e^{8r}$$

$$\ln(0.5) = 8r$$

$$r = \frac{\ln 0.5}{8} \simeq -0.0866$$

($r<0$ means the iodine-(3) decreases over time)

$t = ?$ when $P = 0.01P_0$

$$P = P_0 e^{-0.0866t}$$

$$0.01\cancel{P_0} = \cancel{P_0}e^{-0.0866t}$$

$$0.01 = e^{-0.0866t}$$

$$\ln 0.01 = -0.0866t$$

$$t = \frac{\ln 0.01}{-0.0866} \simeq \boxed{53.2 \text{ days}}$$

Ex 4: The tsunami of 2004 killed over 200,000 people and was measured at M = 9.1 on the Richter Scale. What was its intensity?

(Use $M = \log\left(\frac{I}{I_0}\right)$ where $I_0 = 10^{-3}$ is the zero level earthquake, or the minimum intensity that can be felt.)

$$9.1 = \log\left(\frac{I}{10^{-3}}\right)$$

$$10^{9.1} = \frac{I}{10^{-3}}$$

$$I = 10^{9.1}(10^{-3})$$

$$= 10^{6.1} \simeq 1{,}258{,}925.4$$

Ex 5: Anneke puts \$350 per month into an investment account to save for her retirement. The account earns 6% interest compounded monthly and the account grows according to this formula

$$S(t) = \frac{300((1.005)^{12t} - 1)}{0.005}$$

where t is the number of years she makes the deposits.

How many years must she make monthly deposits in order to have \$1,200,000 in this retirement account?

$S = 1{,}200{,}000 \quad t = ?$

$$1{,}200{,}000 = \frac{300(1.005^{12t} - 1)}{0.005}$$

$$1{,}200{,}000 = 60000(1.005^{12t} - 1)$$

$$20 = 1.005^{12t} - 1$$

$$21 = 1.005^{12t}$$

$$\ln 21 = 12t \ln 1.005$$

$$\frac{\ln 21}{12 \ln 1.005} = t$$

$$t \simeq 50.87 \text{ yrs} \simeq \boxed{51 \text{ yrs}}$$