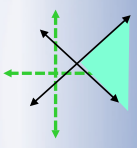
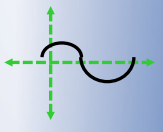
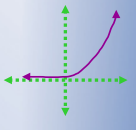
$5x-2y \leq 75$



$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$S = Pe^{rt}$



$APY = (1+\frac{r}{n})^{n}-1$

# Math 1090 ~ Business Algebra

## Section 5.1 Arithmetic and Geometric Sequences

Objectives:

- Distinguish between arithmetic and geometric sequences.
- Recognize a sequence in recursive form and in iterative form.
- Find the nth term of a sequence.
- Find the sum of n terms of a sequence.

Vocabulary

$a_n = f(n)$

Sequence: $\{a_n\}$ an ordered list of numbers that form a pattern. It's also a function with domain of natural numbers.

$n = 1, 2, 3, \ldots$

$a_1\ a_2\ a_3\ a_4\ a_5$

ex 1, 4, 7, 10, 13, ...

Arithmetic Sequence

$a_1 = 1$, $d = 3$

$n = 2,3,\ldots.$ $d \neq 0$

$a_n = a_{n-1} + d$ given $a_1$

(Add the same number to get each of the next terms.)

ex 1, 3, 9, 27, 81, 243, ...

$a_1\ a_2\ a_3\ a_4\ a_5\ a_6$

Geometric Sequence

$a_1 = 1$, $d = 3$

$n = 2,3,\ldots.$ $d \neq 0$

$a_n = da_{n-1}$ given $a_1$

(Multiply by the same number to get each of the next terms.)

* These formulas are recursive. (They depend on previous terms.)

$a_1$ given

$a_2 = a_1 + d$

$a_3 = a_2 + d = (a_1 + d) + d$

$a_4 = a_3 + d = a_1 + 2d + d = a_1 + 3d$

$a_5 = a_4 + d = a_1 + 4d$

$a_6 = a_1 + 5d$

$a_7 = a_1 + 6d$

arith. seq.

$a_n = a_1 + (n-1)d$

$a_1$ given

$a_2 = a_1 d$

$a_3 = a_2 d = (a_1 d)d = a_1 d^2$

$a_4 = a_3 d = (a_1 d^2)d = a_1 d^3$

$a_5 = a_4 d = a_1 d^4$

$a_6 = a_1 d^5$

$a_7 = a_1 d^6$

$a_n = a_1 d^{n-1}$

geom. seq.

* These formulas are iterative. (They don't depend on previous terms.)

(aka direct)

Ex 1: Classify as arithmetic or geometric and give the next three terms of each sequence.

a) 10, 7, 4, 1, . . .

-3 -3 -3

arithmetic seq.
$d = -3$
next terms: -2, -5, -8

b) 2, -6, 18, -54, . . .

x-3 x-3 x-3

geometric seq.
w/ $d = -3$
next terms:
162, -486, 1458

Ex 2: Find a formula for the nth term of each of these.

a) an arithmetic sequence where $a_1 = 2$ and $d = -3$

n=1 n=2 n=3 n=4 n=5

2, -1, -4, -7, -10, . . .

$a_n = a_1 + (n-1)d$

$a_n = 2 + (n-1)(-3)$

$a_n = 2 - 3n + 3$

$a_n = 5 - 3n$

b) a geometric sequence where $a_1 = -10$ and $d = 2$

-10, -20, -40, -80, -160, . . .

$a_n = a_1 d^{n-1}$

$a_n = -10(2^{n-1})$

WARNING:

$-10(2^{n-1}) \neq -20^{n-1}$

remember order of operations, namely that exponents are evaluated before multiplication!

Ex 3: Given $a_1 = 2$ and $a_8 = 23$, find the 50th term of this arithmetic sequence.

① $a_n = a_1 + (n-1)d$

$a_n = 2 + (n-1)d$

plug in $n = 8$:

$23 = 2 + (8-1)d$

$21 = 7d$

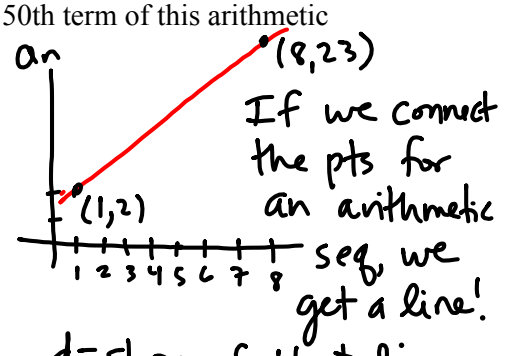
$d = 3$

$\Rightarrow a_n = 2 + (n-1)(3) = 3n - 1$

$a_{50} = 2 + (50-1)(3)$

$= 149$

②




If we connect the pts for an arithmetic seq, we get a line!

d = slope of that line

(1, 2) (8, 23)

$m = \frac{23-2}{8-1} = \frac{21}{7} = 3$

$y - 2 = 3(x-1)$

$y = 3x - 1$

$\Rightarrow a_n = 3n - 1$

Ex 4: Given $a_1 = \frac{3}{2}$ and $a_6 = \frac{3}{64}$, find the 20th term of the geometric sequence.

$a_n = a_1 d^{n-1}$

$a_n = \frac{3}{2} d^{n-1}$

$n = 6$: $a_6 = \frac{3}{2}(d^{6-1})$

$\frac{2}{3}\left(\frac{3}{64}\right) = \left(\frac{3}{2}d^5\right)\frac{2}{3}$

$32$

$\frac{1}{32} = d^5$

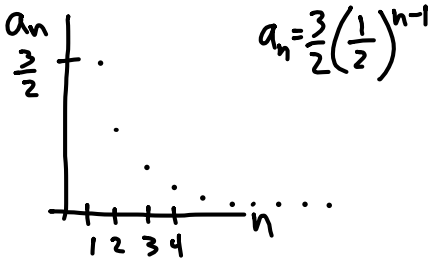
$\sqrt[5]{\frac{1}{32}} = d$

$d = \frac{1}{2}$

$\Rightarrow a_n = \frac{3}{2}\left(\frac{1}{2}\right)^{n-1}$

$a_{20} = \frac{3}{2}\left(\frac{1}{2}\right)^{19}$

$= \frac{3}{2^{20}} = \frac{3}{1{,}048{,}576}$




geom. seq, if we connect the dots, graphs into exponential curve

## Arithmetic Sequence Sum

$S_{12} = 2+5+8+11+14+17+20$
$+23+26+29+32+35 = ?$ $d=3$

($S_{12}$ = sum of 1st 12 terms of the sequence)

$S_{12} = 2+5+8+\ldots+32+35$
$+$
$S_{12} = 35+32+\ldots+5+2$

$2S_{12} = 37+37+37$
$+\ldots+37+37$

$S_{12} = \frac{37(12)}{2}$

$\Rightarrow \boxed{S_n = \frac{(a_1+a_n)n}{2}}$ nth sum of arith. seq

## Geometric Sequence Sum

$S_n = a_1 + a_2 + \ldots + a_n = ?$

remember $a_i = a_1(d^{i-1})$

$S_n = a_1 + a_1d + a_1d^2 + \ldots + a_1d^{n-1}$
$-dS_n = a_1d + a_1d^2 + a_1d^3 + \ldots + a_1d^{n-1} + a_1d^n$

$S_n - dS_n = a_1 - a_1d^n$

$S_n(1-d) = a_1(1-d^n)$

$\boxed{S_n = \frac{a_1(1-d^n)}{1-d}}$ nth sum of geom. seq.

Ex 5: Find the sum of the first $n$ terms of each of these.

a) 1, 10, 19, 28, . . . $n = 100$

+9 +9 +9

arithmetic sequence.

$d=9,\ a_1=1,\ a_{100} = a_1 + 99d$
$= 1+99(9)$
$= 892$

$S_{100} = \frac{(1+892)100}{2}$

$= \frac{89300}{2} = 44{,}650$

b) 3, 6, 12, . . . $n = 10$

×2 ×2

geometric seq.

$d=2,\ a_1=3,\ n=10$

$S_{10} = \frac{3(1-2^{10})}{1-2}$

$= \frac{3(1-1024)}{-1}$

$= 3(1023)$

$= 3069$