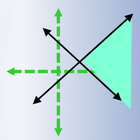
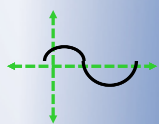
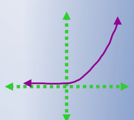
$5x-2y \le 75$



$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$S = Pe^{rt}$



$APY = (1+\frac{r}{n})^n - 1$

Math 1090 ~ Business Algebra

Section 5.2 Simple and Compound Interest

Objectives:

- Differentiate between simple and compound interest.
- Solve problems involving simple and compound interest.
- Understand and calculate annual percentage yield (APY).

Simple and Compound Interest

Simple Interest (like arithmetic seq)

- add same interest every period
- arithmetic sequence (grows linearly)
- balance is the sum

- P = principal = start value
- Pr = principal times interest rate

$$S = P + \Pr(t)$$

$$\boxed{S = P(1 + rt)}$$ simple interest formula

P = principal

r = annual interest rate

t = number of years

S = future account value (acct value after t yrs)

Compound Interest (like a geometric seq)

- multiply by same rate every period
- geometric sequence (grows exponentially)
- balance is the sum

- P = principal = start value
- $(1+ r)$ = factor that's multiplied by principal every year

A $$\boxed{S = P(1 + r)^t}$$ compound interest formula

B If we compound n times per year,

$$S = P\left(1 + \frac{r}{n}\right)^{nt}$$

C Continuous compounding

$$S = Pe^{rt}$$

Ex 1: If \$10,000 is invested for four years at an annual rate of 8%, how much will the account be worth at the end of four years?

a) simple interest

$P = \$10,000$

$t = 4$

$r = 0.08$

$S = P(1+rt)$

$S = 10000(1+0.08(4))$

$= \$13,200$

b) compounded once a year

$S = P\left(1+\frac{r}{n}\right)^{nt}$

$n = 1$

$S = P(1+r)^t$

$S = 10000(1+0.08)^4$

$\approx \$13,604.89$

Ex 2: What is an account worth in 8 years if we started with \$3000 and we got continuous compounding at a rate of 6%?

$P = \$3000$

$r = 0.06$

$t = 8$

$S = Pe^{rt}$

$S = 3000e^{0.06(8)} \simeq \$4848.22$

Ex 3: If \$1000 is invested at 5% annual interest rate, compute these.

$P = 1000$

$r = 0.05$

$S = \$2000$

	balance after 5 years $t = 5$	how long to double investment
simple interest	$S = P(1+rt)$ $S = 1000(1+0.05(5))$ $= \$1250$	$2000 = 1000(1+0.05t)$ $2 = 1+0.05t$ $1 = 0.05t$ $\boxed{t = 20 \text{ yrs}}$
compound interest, $n = 1$	$S = P(1+r)^t$ $S = 1000(1+0.05)^5$ $\simeq \$1,276.28$	$2000 = 1000(1+0.05)^t$ $2 = (1.05)^t$ $\ln 2 = \ln 1.05^t = t \ln 1.05$ $t = \frac{\ln 2}{\ln 1.05} \simeq \boxed{14.2 \text{ yrs}}$
compound interest, $n = 12$	$S = P\left(1+\frac{r}{n}\right)^{nt}$ $S = 1000\left(1+\frac{0.05}{12}\right)^{12(5)}$ $\simeq \$1,283.36$	$2000 = 1000\left(1+\frac{0.05}{12}\right)^{12t}$ $2 = 1.0041\overline{6}^{12t}$ $\ln 2 = 12t \ln 1.0041\overline{6}$ $t = \frac{\ln 2}{12 \ln 1.0041\overline{6}} \simeq \boxed{13.9 \text{ yrs}}$

Compound Continuously

$S = Pe^{rt}$

$S = 1000e^{0.05(5)}$

$\simeq \$1284.03$

$2000 = 1000e^{0.05t}$

$2 = e^{0.05t}$

$\ln 2 = 0.05t$

$t = \frac{\ln 2}{0.05} \simeq \boxed{13.86 \text{ yrs}}$

Ex 4: What amount must be invested now in order to have \$1,000,000 for retirement in 45 years if money is compounded quarterly at 9%?

$$S = P\left(1+\frac{r}{n}\right)^{nt}$$

$t = 45$ yrs

$n = 4$

$r = 0.09$

$S = 1{,}000{,}000$

$P = ?$

$$1000000 = P\left(1+\frac{0.09}{4}\right)^{4(45)}$$

$$1000000 = P(1.0225)^{180}$$

$$P = \frac{1{,}000{,}000}{1.0225^{180}} \simeq \$18{,}222.29$$

APY (Annual Percentage Yield)

Let P = \$100 be invested at 8% interest compounded as given in (a) and (b). What is the account worth after one year?

a) quarterly $n = 4$ $t = 1$ $r = 0.08$ $P = 100$

$$S = P\left(1+\frac{r}{n}\right)^{nt}$$

$$S = 100\left(1+\frac{0.08}{4}\right)^{4(1)}$$

$$\simeq \$108.24$$

$\Rightarrow$ APY = 8.24%

APR = r = 8%

b) monthly $n = 12$

$$S = P\left(1+\frac{r}{n}\right)^{nt}$$

$$S = 100\left(1+\frac{0.08}{12}\right)^{12(1)}$$

$$\simeq \$108.30$$

APY = 8.3%

APR = r = 8%

$$APY = \left(1+\frac{r}{n}\right)^n - 1 \qquad \text{(periodic compounding)}$$

$$APY = e^r - 1 \qquad \text{(continuous compounding)}$$

Ex 5: Which is a better investment deal?

a) 10% compounded annually $n=1,\ r=0.1$

$$APY = \left(1+\frac{0.1}{1}\right)^1 - 1 = 0.1 = 10\%$$

b) 9.8% compounded quarterly $n=4,\ r=0.098$

$$APY = \left(1+\frac{0.098}{4}\right)^4 - 1 \simeq 0.10166 \simeq \boxed{10.166\%} \text{ best deal}$$

c) 9.65% compounded continuously $r=0.0965$

$$APY = e^{0.0965} - 1 \simeq 0.10131 = 10.131\%$$