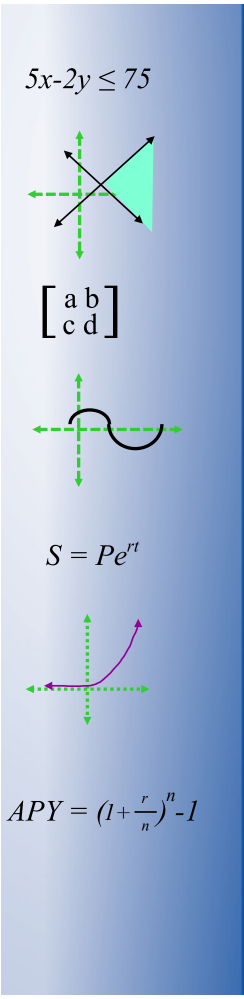

Math 1090 ~ Business Algebra

Section 5.4 Present Value of Annuities

Objectives:

- Determine the present value of an ordinary annuity.
- Solve problems involving annuities.
- Distinguish between present value and future value word problems.

Present Value of an annuity: We calculate this when we leave a lump sum of dollars in an account and make regular withdrawals (like what happens after a person retires.)

ordinary annuity	annuity due
withdrawals occur at the end of each period.	withdrawals occur at the beginning of each period.

Ex 1: You want to withdraw \$1000 at the end of each year from an account that earns 10% interest compounded annually for 4 years. How much needs to be in the account from the start?

$$S = P\left(1+\frac{r}{n}\right)^{nt} \qquad r_c = \frac{r}{n} \qquad N = nt \qquad S = P(1+r_c)^N$$

compound interest formula

$\underset{(FV)}{S} = 1000, \quad r_c = 0.1, \quad n = 1 \qquad P = \dfrac{S}{(1+r_c)^N} = S(1+r_c)^{-N}$

After 1st year: $N=1$ $\quad P_1 = 1000(1.1)^{-1} \approx \$909.09$

After 2nd year: $N=2$ $\quad P_2 = 1000(1.1)^{-2} \approx \$826.45$

After 3rd year: $N=3$ $\quad P_3 = 1000(1.1)^{-3} \approx \$751.31$

After 4th year: $N=4$ $\quad P_4 = 1000(1.1)^{-4} \approx \$683.01$

$\Rightarrow$ today, in the acct, I need lump sum of

$$P_1+P_2+P_3+P_4 = 1000(1.1)^{-1}+1000(1.1)^{-2}+1000(1.1)^{-3}+1000(1.1)^{-4}$$

this is sum of geom. sequence again!!!

Present Value of an Ordinary Annuity	Present Value of an Annuity Due
$P = R\left[\frac{1-(1+r_c)^{-N}}{r_c}\right]$	$P_{due} = \left[\frac{R(1+r_c)(1-(1+r_c)^{-N})}{r_c}\right]$

Ex 2: Find PV of an annuity that pays \$4000 at the end of each month from an account that earns 8% interest compounded monthly for 25 years.

PV ordinary annuity

(PV = lump sum amt. in acct. to produce all these payments)

$R=4000,\ t=25,\ r=0.08,\ n=12,\ r_c=\frac{0.08}{12}=0.00\overline{6}$

$P=4000\left(\frac{1-1.00\overline{6}^{-300}}{0.00\overline{6}}\right)\approx\boxed{\$518{,}258.09}$ $N=12(25)=300$

total withdrawals from acct:

$4000(25)(12)=\$1{,}200{,}000$

Ex 3: An inheritance of \$500,000 will provide how much at the end of each year for 20 years if money is worth 7.2% compounded annually?

$n=1,\ t=20,\ r=0.072$

$r_c=0.072,\ N=20$

(lump sum in present)

PV ordinary annuity:

$P=R\left[\frac{1-(1+r_c)^{-N}}{r_c}\right]$

$R=?$

$P=\$500{,}000$

$500{,}000=R\left[\frac{1-1.072^{-20}}{0.072}\right]$

$R=500{,}000\left[\frac{0.072}{1-1.072^{-20}}\right]\approx\boxed{\$47{,}932.61}$

total withdrawals:

$47932.61(1)(20)=\$958{,}652.20$

Deferred Annuity: The first payment is deferred until a later date at which point regular payments are made.

P = PV of deferred annuity

m = number of periods of deferment

N = number of regular withdrawals

R = payment each period

$$\boxed{P = \frac{R(1-(1+r_c)^{-N})}{r_c(1+r_c)^m}}$$

Ex 4: Carol received a trust fund inheritance of \$10,000 on her 30th birthday. She plans to use it to supplement her income with 20 quarterly payments beginning on her 60th birthday. If money is worth 8.1% compounded quarterly, how much will each payment be?

$r = 0.081$, $n = 4$, $P = 10{,}000$

$r_c = \frac{0.081}{4} = 0.02025$, $N = 20$, $m = 30(4) = 120$

$$10{,}000 = \frac{R(1-1.02025^{-20})}{0.02025(1.02025^{120})}$$

$$R = \frac{10{,}000(0.02025(1.02025^{120}))}{(1-1.02025^{-20})}$$

$R \simeq \boxed{\$6{,}796.47}$

total withdrawals:

$6796.47(20) = \$135{,}929.40$

Ex 5: A lottery prize worth \$1,800,000 is awarded in payments of \$10,000 at the beginning of each month for 15 years. Suppose money is worth 6.6% monthly. What is the real value of the prize?

$n = 12$

$R = 10000$

$r = 0.066$

$r_c = \frac{0.066}{12} = 0.0055$

$t = 15$

$N = 15(12) = 180$

total payments:

$10{,}000(15)(12) = \$1{,}800{,}000$ ✓

$$PV_{due} = \frac{R(1+r_c)(1-(1+r_c)^{-N})}{r_c}$$

$$= \frac{10000(1.0055)(1-1.0055^{-180})}{0.0055}$$

$$\approx \boxed{\$1{,}147{,}026.90}$$