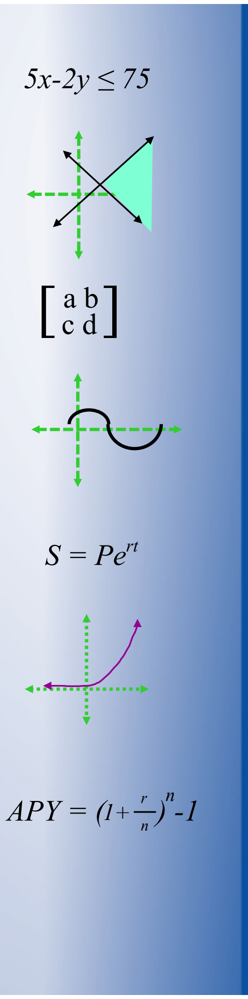


# Math 1090 ~ Business Algebra

## Section 5.5 Loans and Amortization

Objectives:

- Create an amortization schedule for a loan.
- Determine the amount of a loan you can afford given certain conditions.
- Determine the amount of each monthly payment for a given number of years.

<u>Amortization</u>: An "installment loan" is a loan that is repaid by making all payments equal.

The bank is basically investing a lump sum of dollars and getting a periodic return which is exactly like PV of an ordinary annuity.

$$\boxed{R = S\left(\frac{r_c}{1-(1+r_c)^{-N}}\right)}$$ Amortization Formula

notice this is just the PV (ord.) formula rearranged to get R by itself!

S = loan amount R = payment amount

Ex 1: When you graduate college, you buy a new car and can afford a monthly payment of $250/month. If you get a special rate of 3.6% interest, compounded monthly, for 6 years, how much can you afford to borrow?

$n=12$

$R = \$250,\ r = 0.036,\ t = 6 \text{ yrs},\ S = ?$

$r_c = \frac{0.036}{12} = 0.003,\ N = 6(12) = 72$

$$250 = S\left(\frac{0.003}{1-1.003^{-72}}\right)$$

$$S = 250\left(\frac{1-1.003^{-72}}{0.003}\right)$$

$$S \simeq \boxed{\$16,167.01}$$

total payments:

$250(72) = \$18,000$

Ex 2: Alex buys a house for $200,000. They put $15,000 down and get a loan for the rest at 5.4% interest compounded monthly for 20 years. What will their payments be?

$S = \$185{,}000, \; r = 0.054, \; n = 12, \; t = 20\text{yrs}$

$r_c = \frac{0.054}{12} = 0.0045, \; N = 12(20) = 240$

$$R = 185000\left(\frac{0.0045}{1 - 1.0045^{-240}}\right)$$

$R \approx \boxed{\$1262.17}$

total payments:
$1262.17(240) = \$302{,}920.80$

Amortization Schedule

A loan of $10,000 with interest rate of 10% could be repaid in 5 equal annual payments.

$$R = S\left(\frac{r_c}{1-(1+r_c)^{-N}}\right)$$

$r = 0.1, \; n = 1, \; t = 5$

$r_c = 0.1, \; N = 1(5) = 5$

$R = 10000\left(\frac{0.1}{1 - 1.1^{-5}}\right) \approx \$2637.97$

int + principal = pymnt

| | payment | interest | principal | unpaid balance |
|---|---|---|---|---|
| 1 | 2637.97 | 10000(0.1) = 1000 | 1637.97 | 8362.03 |
| 2 | 2637.97 | 8362.03(0.1) = 836.26 | 1801.77 | 6560.26 |
| 3 | 2637.97 | 6560.26(0.1) = 656.03 | 1981.94 | 4578.32 |
| 4 | 2637.97 | 4578.32(0.1) = 457.83 | 2180.14 | 2398.18 |
| 5 | 2637.97 | 2398.18(0.1) = 239.82 | 2398.15 | 0.03 |

= 10,000 − 1637.97

= 8362.03 − 1801.77

= 6560.26 − 1981.94

= 4578.32 − 2180.14

= 2398.18 − 2398.15

⇒ your very last payment will actually be $2638.00 (to cover that leftover 3¢)

total payments:
$2637.97(5) = \$13{,}189.85$

Ex 3: A company that buys a piece of equipment by borrowing \$250,000 for 10 years at 6% compounded monthly has monthly payments of \$2,775.51.

$t = 10$, $r = 0.06$, $n = 12$

$r_c = 0.005$

$N = 120$

$R = \$2775.51$

a) Find the unpaid balance after 1 year.

$k = 12 \quad N-k = 108$

$$S_{108} = 2775.51\left(\frac{1 - 1.005^{-108}}{0.005}\right)$$

$$S_{108} \approx \$231,181.73$$

b) During that first year, how much interest does the company pay?

$2775.51(12) = \$33,306.12$ (total payments in $1^{st}$ year)

$\Rightarrow 250,000 - 231,181.73 = \$18,818.27$

(amt that went toward principal)

$\Rightarrow 33,306.12 - 18,818.27 = \boxed{\$14,487.85}$

(this is the amt of $1^{st}$ year of payments that went toward interest)

**Loan Payoff Amount**

$$S_{N-k} = R\left[\frac{1-(1+r_c)^{-(N-k)}}{r_c}\right]$$

$k$ = number of payments that have been made.

$N = nt$ = total number of payments that were originally due.

$N$-$k$ = number of payments "missing" from the loan.