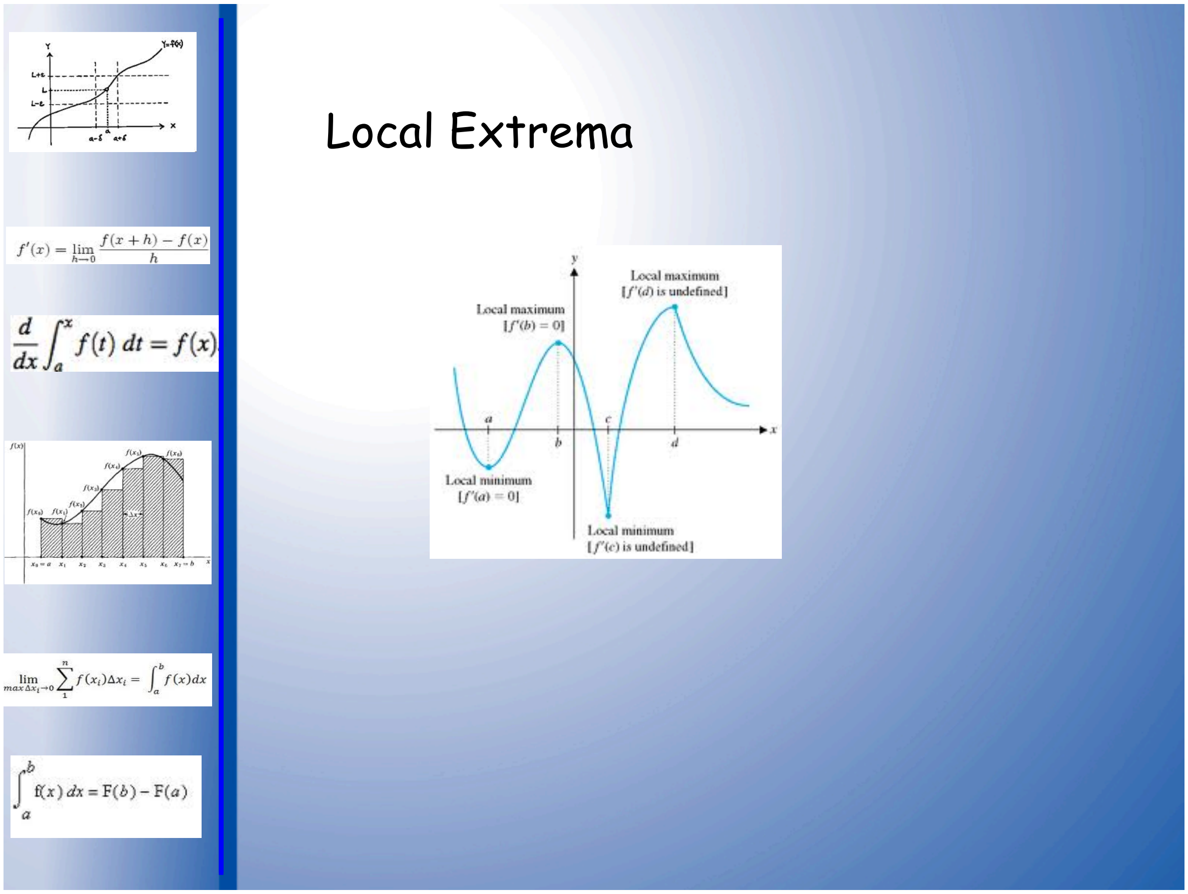
18 Local Extrema



Definition

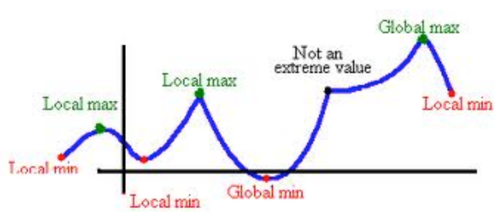
Let *S* be the domain of *f* such that *c* is an element of *S*.

Then,

1) *f(c)* is a **local maximum** value of *f* if there exists an interval *(a,b)* containing *c* such that *f(c)* is the maximum value of *f* on $(a,b)\cap S$.

2) *f(c)* is a **local minimum** value of *f* if there exists an interval *(a,b)* containing *c* such that *f(c)* is the minimum value of *f* on $(a,b)\cap S$.

3) f(c) is a **local extreme value** of *f* if it is either a local maximum or local minimum value.



18 Local Extrema

How do we find the local extrema?

First Derivative Test

Let *f* be continuous on an open interval *(a,b)* that contains a critical *x-value*.

1) If $f'(x) > 0$ for all x on (a,c) and $f'(x)<0$ for all x on (c,b), then $f(c)$ is a local maximum value.
2) If $f'(x) < 0$ for all x on (a,c) and $f'(x)>0$ for all x on (c,b), then $f(c)$ is a local maximum value.
3) If f'(x) has the same sign on both sides of c, then *f(c)* not a maximum nor a minimum value.

EX 1 Determine local maximum and minimum points for $y = 2x^2 - 5x + 3$.

EX 2 Find all local maximum and minimum points for $f(x) = \frac{1}{2}x + sinx \ on \ [0, 2\pi]$.

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Theorem: Second Derivative Test

Let f' and f'' exist at every point on the interval (a,b) containing c and $f'(c) = 0$.

1) If $f''(c) < 0$, then $f(c)$ is a local maximum.
2) If $f''(c) > 0$, the $f(c)$ is a local minimum.

EX 3 Find all critical points for $f(x) = x^3 - 3x^2 + 1$.

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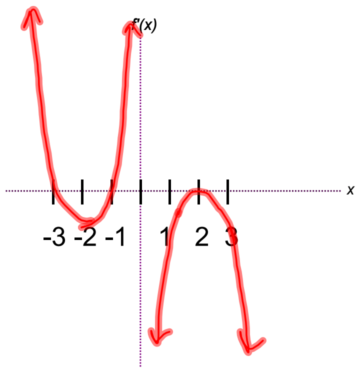
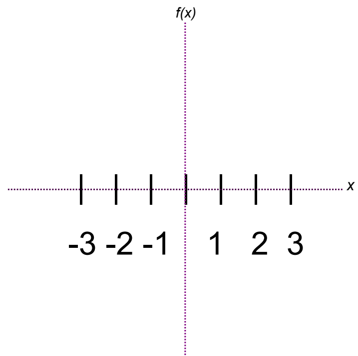
EX 4 Find local and global extrema for $y = x^2 + \frac{1}{x^2} \; on \; [-2, 2]$.

18 Local Extrema

EX 5 Let f be continuous such that f' has the following graph.

Try to sketch a graph of $f(x)$ and answer these questions.





a) Where is f increasing?

b) Where is f decreasing?

c) Where is f concave up?

d) Where is f concave down?

e) Where are inflections points?

f) Where are local max/min values?