Math 1210 #18
Local Extrema

Definition

Let S be the domain of f such that c is an element of S.
Then,

1. $f(c)$ is a **local maximum** value of f if there exists an interval (a, b) containing c such that $f(c)$ is the maximum value of f on $(a, b) \cap S$.
2. $f(c)$ is a **local minimum** value of f if there exists an interval (a, b) containing c such that $f(c)$ is the minimum value of f on $(a, b) \cap S$.
3. $f(c)$ is a **local extreme value** of f if it is either a local maximum or local minimum value.

How do we find the local extrema?

First Derivative Test

Let f be continuous on an open interval (a, b) that contains a critical x-value.

1. If $f'(x) > 0$ for all x on (a, c) and $f'(x) < 0$ for all x on (c, b), then $f(c)$ is a local maximum value.
2. If $f'(x) < 0$ for all x on (a, c) and $f'(x) > 0$ for all x on (c, b), then $f(c)$ is a local maximum value.
3. If $f'(x)$ has the same sign on both sides of c, then $f(c)$ not a maximum nor a minimum value.

EX 1

Determine local maximum and minimum points for $y = 2x^2 - 5x + 3$.

EX 2

Find all local maximum and minimum points for $f(x) = \frac{1}{2}x + \sin\ x$ on $[0, 2\pi]$.

Theorem: Second Derivative Test

Let f' and f'' exist at every point on the interval (a, b) containing c and $f'(c) = 0$.

1. If $f''(c) < 0$, then $f(c)$ is a local maximum.
2. If $f''(c) > 0$, the $f(c)$ is a local minimum.

EX 3

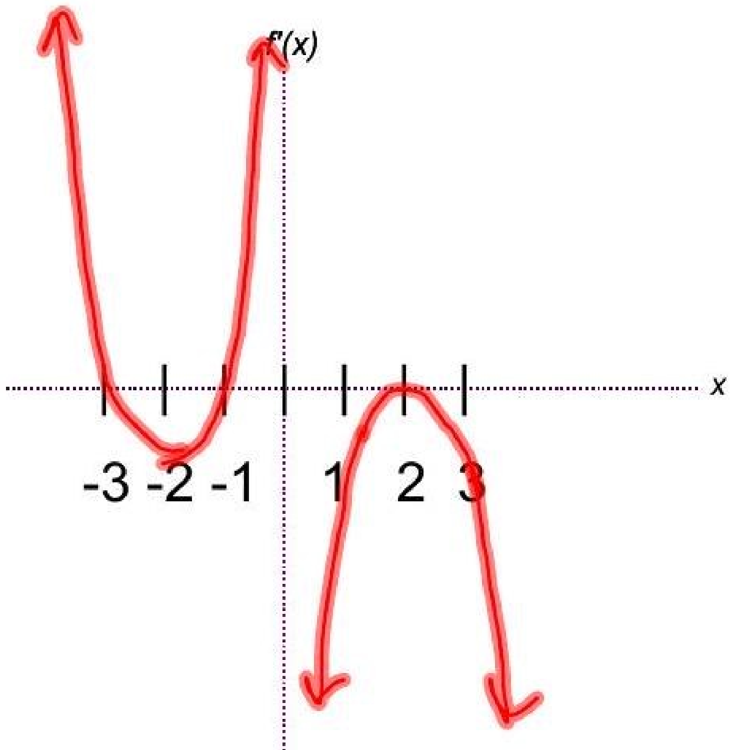
Find all critical points for $f(x) = x^3 - 3x^2 + 1$.

EX 4

Find local and global extrema for $y = x^2 + \frac{1}{x^2}$ on $[-2,2]$.

EX 5

Let f be continuous such that f' has the following graph.
Try to sketch a graph of $f(x)$ and answer these questions.



5a)

Where is f increasing?

5b)

Where is f decreasing?

5c)

Where is f concave up?

5d)

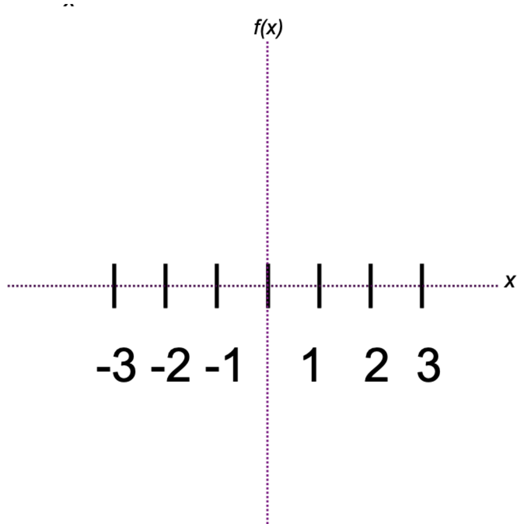
Where is f concave down?

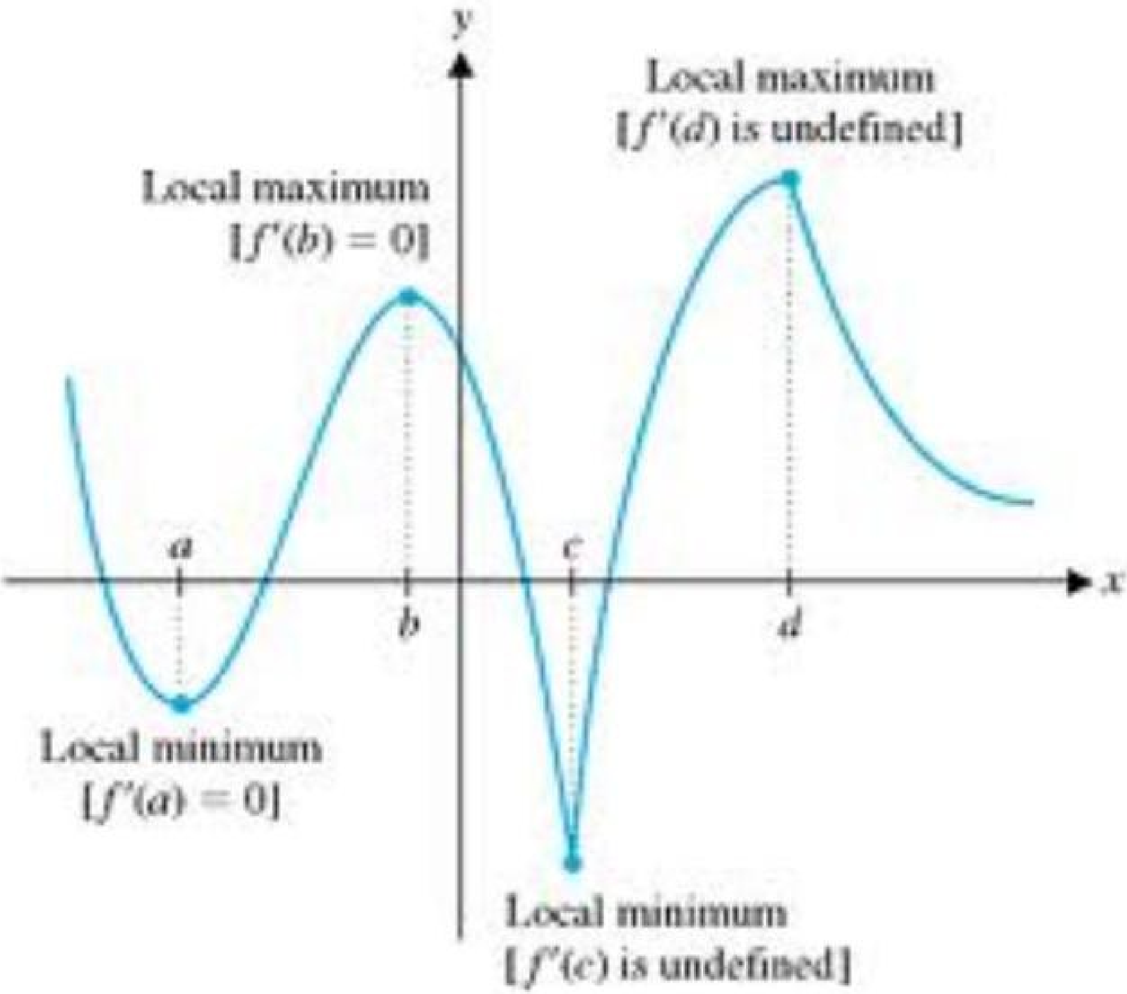
5e)

Where are inflections points?

5f)

Where are local max/min values?




y
Local maximum
[f′(d) is undefined]
Local maximum
[f′(b) = 0]
a
c
x
b
d
Local minimum
[f′(a) = 0]
Local minimum
[f′(c) is undefined]