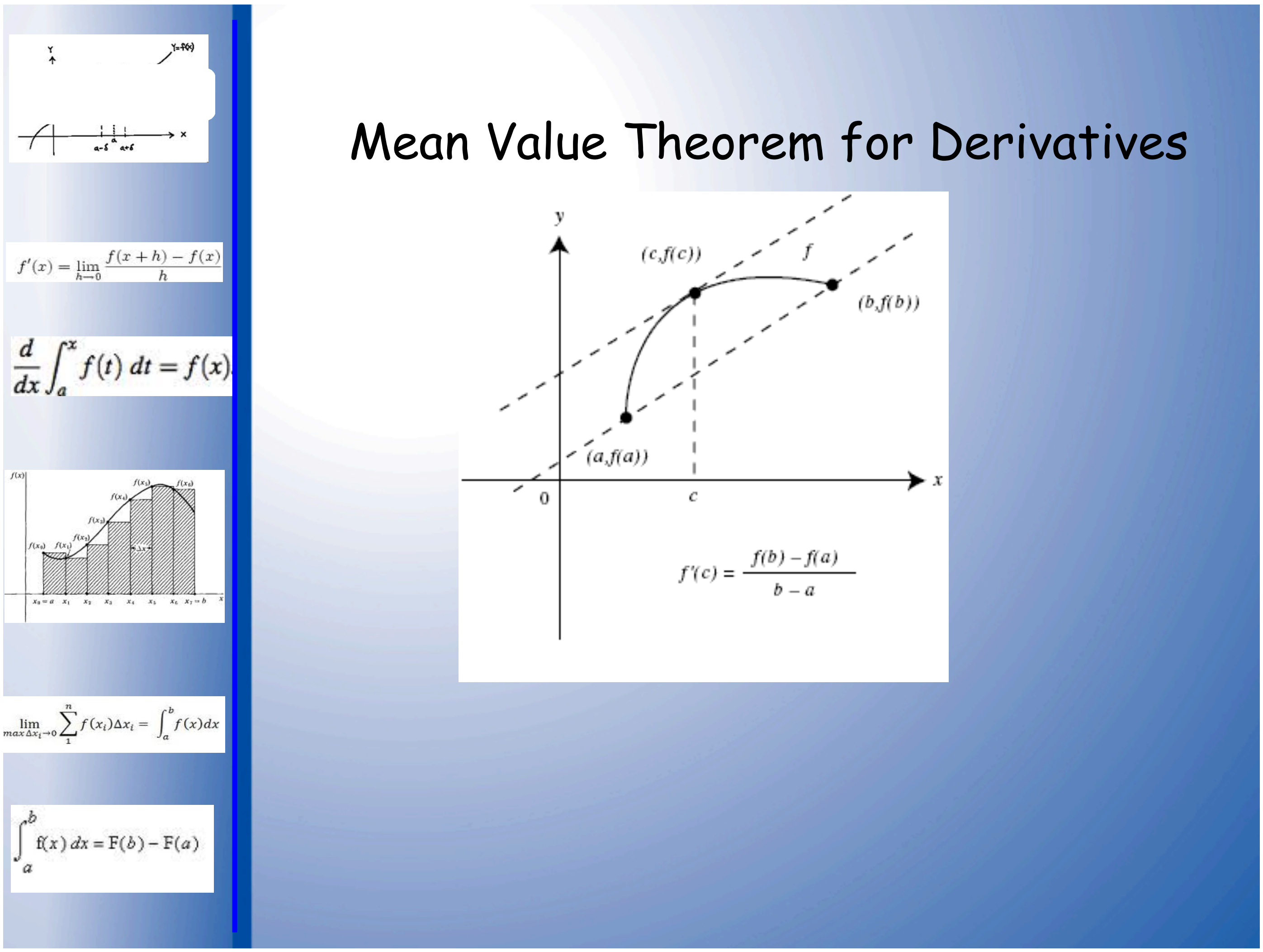
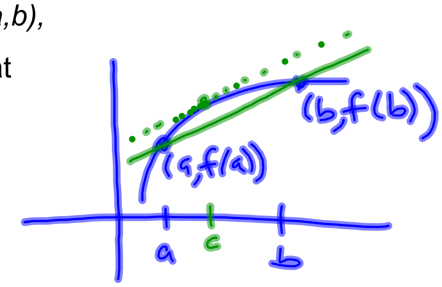

Mean Value Theorem for Derivatives
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$
$\lim_{\max \Delta x_i \to 0} \sum_1^n f(x_i) \Delta x_i = \int_a^b f(x) dx$
$\int_a^b f(x)\, dx = F(b) - F(a)$
y
(c,f(c))
f
(b,f(b))
(a,f(a))
0
c
x
$f'(c) = \frac{f(b) - f(a)}{b - a}$

Mean Value Theorem for Derivatives

If f is ① continuous on $[a,b]$ and ② differentiable on (a,b),

then there exists at least one c on (a,b) such that



$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

slope of secant line (thru x=a, x=b pts) = slope tangent line to curve at x=c.

EX 1 Find the number c guaranteed by the MVT for derivatives for

$g(x)=(x+1)^3$ on $[-1,1]$

meet conditions?

① $g(x)$ cont on $[-1,1]$ ✓

② $g'(x)$ is fine on $(-1,1)$ ✓ (exists)

$a=-1,\ b=1$

$g(b)=(1+1)^3=8,\ g(-1)=0$

Secant line slope: $\dfrac{8-0}{1-(-1)}=\dfrac{8}{2}=4$

tangent line slope: $g'(x)=3(x+1)^2$

$$4=3(c+1)^2$$

$$\frac{4}{3}=(c+1)^2$$

$$\pm\frac{2}{\sqrt{3}}=(c+1)$$

$$c=-1\pm\frac{2}{\sqrt{3}}=\frac{-\sqrt{3}\pm 2}{\sqrt{3}}$$

(need c value in $[-1,1]$)

$$\frac{-\sqrt{3}+2}{\sqrt{3}} \simeq \frac{0.3}{\sqrt{3}} \in [-1,1]$$

$$\frac{-\sqrt{3}-2}{\sqrt{3}} \simeq \frac{-3.7}{1.7} < -2 \notin [-1,1] \rightarrow \boxed{c=\frac{-\sqrt{3}+2}{\sqrt{3}}}$$

EX 2 For $g(x)=\frac{x-4}{x-3}$, decide if we can use the MVT for derivatives on *[0,5]* or *[4,6]*. If so, find *c*. If not, explain why.
(1) (2)

$g(x)=\frac{x-4}{x-3}$

domain: $x\in\mathbb{R}$, $x\neq 3$

(1) [0,5] contains the VA at $x=3$
(g(x) not continuous on [0,5])
$\Rightarrow$ MVT for deriv. does not apply!

(2) on [4,6], g(x) continuous & differentiable.
$\Rightarrow$ can use MVT for deriv.

$g(x)=\frac{x-4}{x-3}$ $a=4, b=6$, $g(6)=\frac{2}{3}$, $g(4)=0$

slope of secant line: $\frac{\frac{2}{3}-0}{6-4}=\frac{2/3}{2}=\frac{1}{3}$

slope of tangent $g'(x)=\frac{(x-3)(1)-(x-4)(1)}{(x-3)^2}=\frac{-3+4}{(x-3)^2}=\frac{1}{(x-3)^2}$

$\frac{1}{(c-3)^2}=\frac{1}{3}$

$(c-3)^2=3$

$c-3=\pm\sqrt{3}$

$c=3\pm\sqrt{3}$

need $c\in[4,6]$

$\Rightarrow \boxed{c=3+\sqrt{3}}$

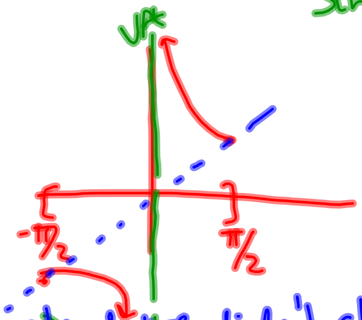
EX 3 For $f(x) = \csc x$ on $[-\pi/2,\ \pi/2]$, use the MVT for derivatives to find *c*.

$f(x) = \dfrac{1}{\sin x}$

notice: $\sin 0 = 0$

$\Rightarrow f(x)$ has discontinuity (VA) at $x = 0$

$x = 0 \in [-\pi/2, \pi/2] \Rightarrow$ we cannot apply MVT for deriv.



pretend we didn't check conditions

$f'(x) = -\csc x \cot x$

secant line: $a = -\pi/2,\ b = \pi/2$

$f(-\pi/2) = -1,\ f(\pi/2) = 1$

slope $= \dfrac{1-(-1)}{\pi/2 - (-\pi/2)} = \dfrac{2}{\pi}$

$-\csc x \cot x = \dfrac{2}{\pi}$

$-\dfrac{\cos x}{\sin^2 x} = \dfrac{2}{\pi}$

$-\cos x = \dfrac{2}{\pi}\sin^2 x$

$-\cos x = \dfrac{2}{\pi}(1-\cos^2 x)$

$\dfrac{2}{\pi}\cos^2 x - \cos x - \dfrac{2}{\pi} = 0$

use quadratic formula to solve for $\cos x$.

⋮

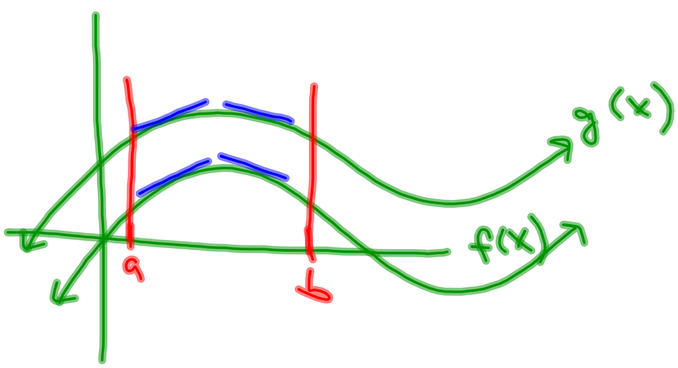
get an actual answer for x.

Theorem B

If $f'(x) = g'(x)$ for all x on the interval (a,b),

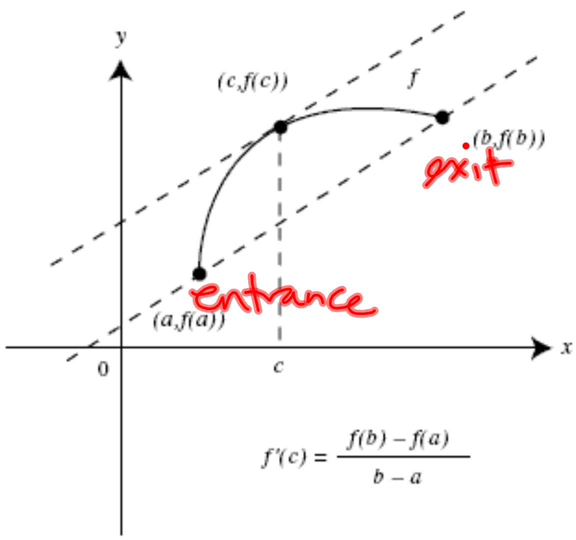
then there exists a *real number, c,* such that $f(x) = g(x) + c$

for all x in the interval (a,b).



at all pts in $[a,b]$, the slopes of curves are the same

note: if $f'(x) = g'(x)$, then $f(x)$ & $g(x)$ curves have same shape



picture for
MVT of derivatives